Metamaterials

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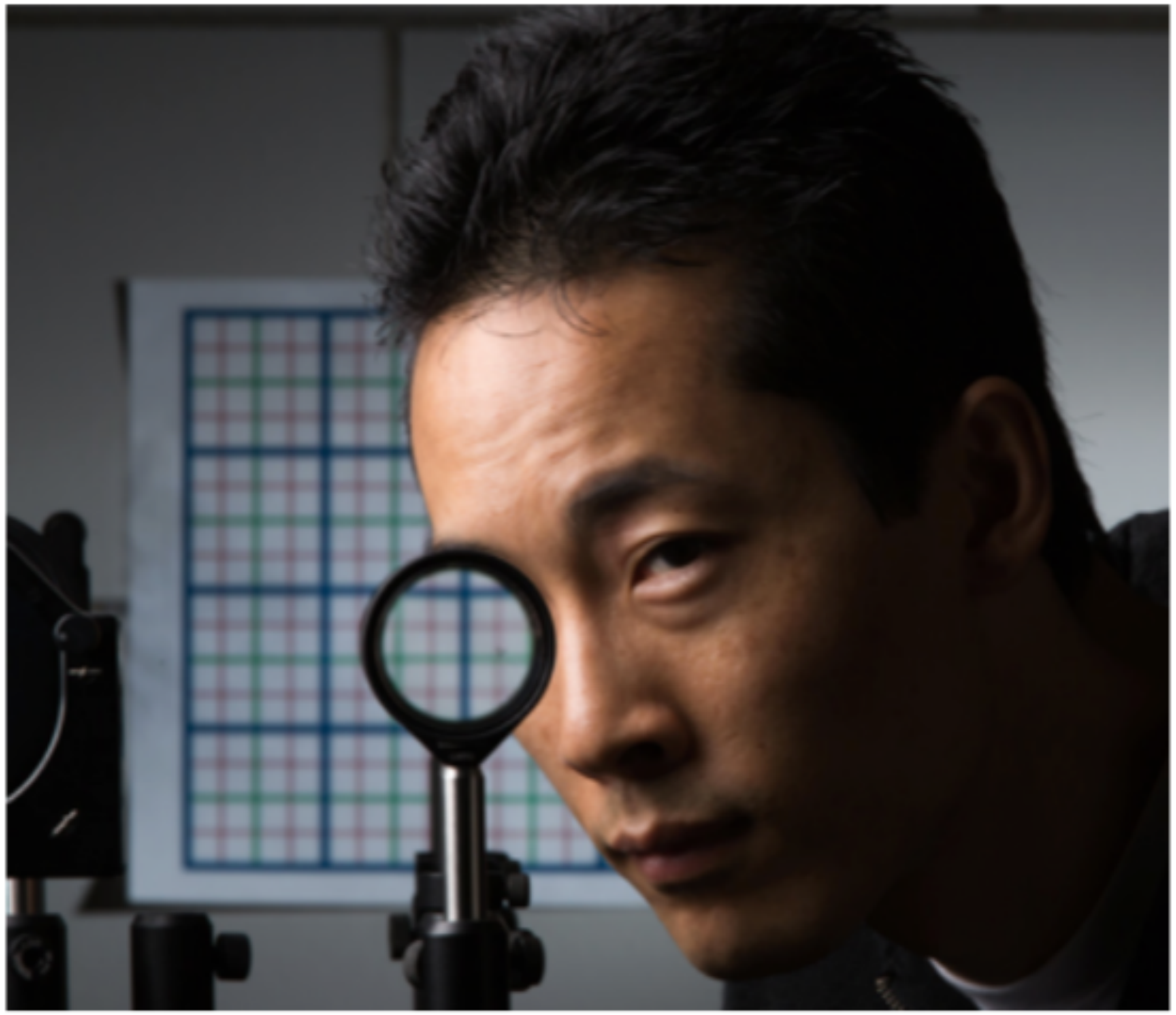

Approches Théoriques pour les Métamatériaux
Quiberon, 11-16 Sept. 2023

Metamaterials

Metamaterials

10th International Congress on Industrial and Applied Mathematics
ICIAM 2023 TOKYO

[00778] Analysis, Applications, and Advances in Metamaterials and Composites

- **Organizer(s)** : Maxence Cassier (CNRS, Institut Fresnel), Graeme W. Milton (University of Utah), Anthony Stefan (Florida Institute of Technology), Aaron Welters (Florida Institute of Technology)

[00932] Some recent advances on time-modulated metamaterials

- **Organizer(s)** : Kshiteej J. Deshmukh, Ornella Mattei

[00239] Shape and Topology Optimizations

- **Organizer(s)** : Takayuki Yamada, Grégoire Allaire, Hideyuki Azegami

Metamaterials

meta Mechanical Acoustic Thermal

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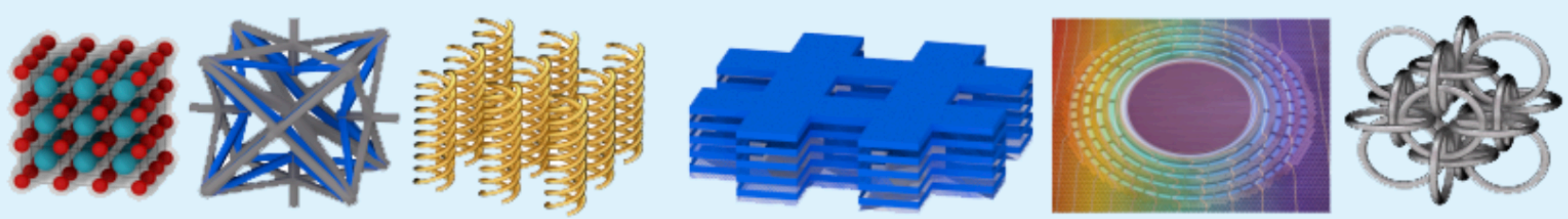

ACADEMIC PARTNERS:

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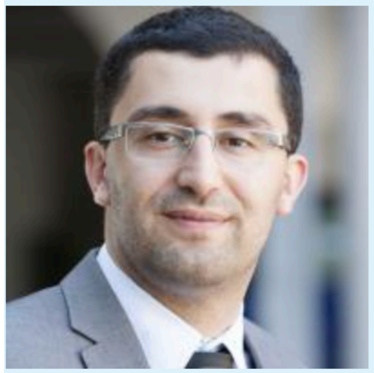
The purpose of this website (META-MAT.ORG) is to disseminate research results in acoustic, thermal and mechanical metamaterials through the organization of online seminars, symposia, conferences, summer schools and workshops with industry and the academic world.

150 séminaires en ligne depuis Juin 2020



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Metamaterials

Outline of Course 1

- Definition of a metamaterial
- Theoretical tool to anticipate the « extraordinary » properties of metamaterials : the asymptotic homogenization
- Asymptotic homogenization of a microstructure in its bulk
- Applications

What is the definition of a metamaterial ?

- a material engineered to have a property that is not found in naturally occurring materials
- a material with extraordinary properties (mechanical, wrt wave propagation)

The scientific activity (mechanics, physics) on metamaterials started in the 2000s

NewScientist

Are we in the Metamaterial Age?

We devise so many new materials nowadays that it is hard to know which one would define our times

TECHNOLOGY 6 March 2013

THE Stone Age; the Bronze Age; the Iron Age. Mastery over a new material has at times changed human society so radically as to be practically synonymous with whole swathes of our history.

Are those times over? Today, we rely more than ever on inventing, rather than discovering, new materials. Precision engineering on microscopic scales even lets us devise metamaterials that behave in ways that nature can't match – like those that manipulate light to make much-celebrated "invisibility cloaks".

Fev. 2020

LES MÉTAMATÉRIAUX EN MÉCANIQUE

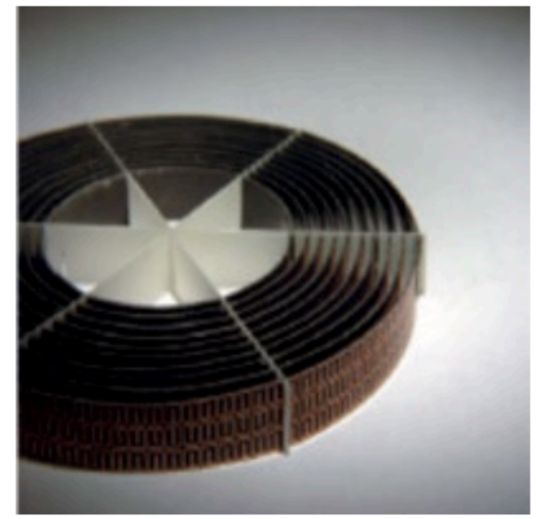
JEAN-JACQUES MARIGO
laboratoire de mécanique des solides (LMS) de l'École polytechnique

Au cours des vingt dernières années, de nouveaux matériaux sont sortis des laboratoires universitaires. Possédant des propriétés électromagnétiques, acoustiques, mécaniques et thermiques que l'on ne rencontre pas dans la nature, ils ont été qualifiés de métamatériaux (*méta* signifiant *au-delà* en grec). Parmi ces propriétés extraordinaires, celle qui à ce jour a le plus frappé l'esprit du grand public est sans nul doute celle, baptisée « cape d'invisibilité », relative à la propagation des ondes électromagnétiques.

On peut relever quelques applications à la mécanique du principe de la cape d'invisibilité. L'idée générale est que, en utilisant différents matériaux ayant des forts contrastes de propriétés et en jouant sur leur agencement géométrique, on peut obtenir des propriétés effectives que l'on n'observe pas dans la nature sur les matériaux réels. L'origine de ce comportement « anormal » peut être simplement statique, mais les propriétés les plus originales sont obtenues en dynamique en s'appuyant sur le phénomène de résonance. Les paragraphes qui suivent vont l'illustrer.

Les matériaux auxétiques

Rappelons que, mis en évidence (analytiquement) par Siméon Denis Poisson, le coefficient dit de Poisson permet de caractériser la contraction de la matière perpendiculairement à la direction de l'effort appliqué. Les matériaux traditionnels ont un coefficient de Poisson positif : quand on les étire dans une direction, ils se

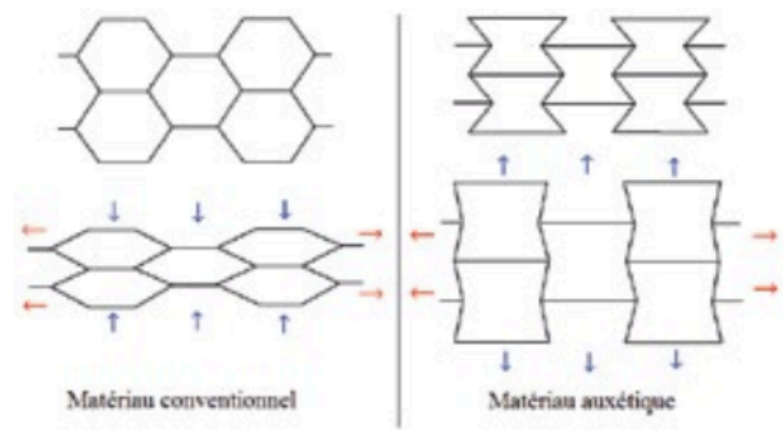


Exemples de cape d'invisibilité. En haut : un « métamatériau » pour les micro-ondes constitué de couches concentriques d'anneaux fendus dont la taille croît avec la distance entre les anneaux et le centre du dispositif (Duke University, Imperial College London) ; en bas : un « métamatériau » pour les vagues constitué de piliers (Institut Fresnel, Marseille).

REPÈRES

Le principe de la « cape d'invisibilité » est le suivant : en arrangeant astucieusement dans une région de l'espace des hétérogénéités (de petite taille), les ondes seront déviées, contourneront l'obstacle et rendront invisible à un observateur extérieur cette région de l'espace. On voit immédiatement toutes les applications que l'on pourrait tirer de ce phénomène. Cette propriété n'est pas spécifique aux ondes électromagnétiques, mais peut se généraliser à tout phénomène ondulatoire. D'où par exemple l'idée de l'Institut Fresnel de fabriquer des amortisseurs de houle en disposant des piliers sur des couronnes concentriques. Ici il s'agit de protéger la région centrale des effets de la houle, les piliers servant de déviateurs.

https://www.lajauneetlarouge.com/wp-content/uploads/2020/01/La_Jaune_et_la_Rouge_751_34-35.pdf



Obtention d'un matériau à coefficient de Poisson négatif à partir de cellules réentrantes (à gauche), alors qu'un agencement en nid-d'abeilles (à droite) donnerait un coefficient de Poisson positif.

contractent dans les directions transverses. De la même façon, si on fabrique une structure composite en disposant un matériau en nid-d'abeilles, le comportement effectif obtenu correspondra à un matériau avec coefficient positif. En revanche, si le matériau de base est disposé suivant des cellules réentrantes (c'est-à-dire concaves), alors on obtient un matériau effectif auxétique, c'est-à-dire à coefficient de Poisson négatif.

La conversion d'ondes par une forêt d'arbres

On envisage aujourd'hui de protéger certains sites sensibles des risques sismiques en plantant une forêt d'arbres de hauteur variable. On s'appuie pour cela sur le fait que les arbres vont modifier la propagation des ondes guidées (Love) ou de surface (Rayleigh) en créant des conditions aux limites effectives à la surface du sol différentes des conditions de bord libre usuelles en l'absence d'arbres. La théorie, étayée par la simulation numérique, montre qu'une onde incidente de surface peut être entièrement réfléchie ou convertie en onde de volume lorsque la fréquence d'excitation correspond à une fréquence propre de vibration d'une rangée d'arbres, par simple effet de résonance. Ces résultats théoriques fondamentaux intéressent évidemment le génie parasismique. Il reste à vérifier la faisabilité du système en vraie grandeur.

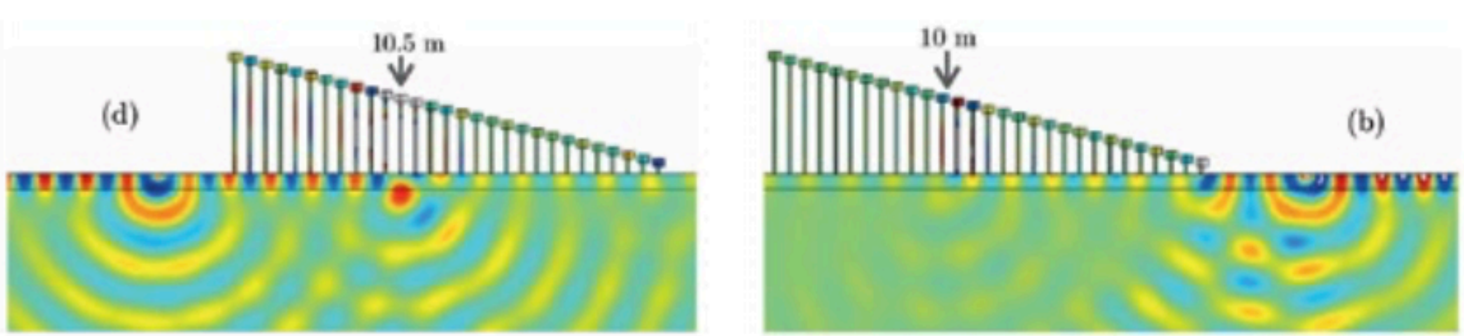
La récupération d'énergie par résonance locale

En utilisant des matériaux élastiques à fort contraste de rigidité, on peut obtenir un matériau dont la masse effective en dynamique devient négative à certaines fréquences. C'est le cas par exemple d'une matrice raide comportant un réseau périodique d'inclusions très molles. On montre, par un calcul explicite utilisant des techniques asymptotiques, que du fait d'un phénomène de résonance locale des inclusions la masse effective de ce composite est négative dans certaines bandes de fréquence. De ce fait les ondes élastiques ne pourront pas se propager dans le composite à ces fréquences-là, qui constituent donc des bandes interdites. Cette propriété peut être utilisée pour concentrer de l'énergie dans une zone prédéfinie d'un microsystème comme l'illustre l'exemple suivant : on insère deux bandes de largeur l d'un métamatériau composite dans la matrice, en les espaçant d'une distance 2d. Si l'on envoie un signal à une fréquence correspondant à une fréquence interdite du métamatériau, alors on peut obtenir une transmission totale dans la partie droite, pourvu que l'espacement d soit convenablement choisi. De plus, l'amplitude du signal dans la partie centrale de la matrice sera considérablement amplifiée. On a donc concentré de l'énergie dans la partie centrale. Ce phénomène est analogue à l'effet tunnel en mécanique quantique. Il s'explique par les divers effets de résonance qui sont présents dans cette situation (dans les métamatériaux et dans la partie centrale).

Des perspectives stimulantes

Les métamatériaux ouvrent donc de formidables perspectives du point de vue des applications. Ils constituent également un grand défi tant scientifique que technologique. D'un point de vue scientifique, il s'agit de mieux comprendre les phénomènes qui conduisent à ces comportements extraordinaires, de bien identifier les microstructures susceptibles de les provoquer et de construire des modèles prédictifs capables d'en rendre compte. Du point de vue technologique, il s'agira d'être capable de fabriquer à grandes échelles ces matériaux microstructurés et de valider sur le terrain leurs performances pressenties.

Perturbation d'une onde de Love (à 70 Hz) par une forêt d'arbres de hauteur variable: (d) conversion en onde de volume d'une onde venant de la gauche; (b) réflexion d'une onde venant de la droite.



Metamaterials

Polytechnique insights
LA REVUE DE L'INSTITUT POLYTECHNIQUE DE PARIS

Invisibilité : nous ne sommes plus très loin du but

Le 23 février 2022 5 min. de lecture

Kim Pham
professeur associé en mécanique à l'ENSTA Paris (IP Paris)

#1 Les ondes acoustiques :

Devenir invisible au sonar

#2 Les ondulations maritimes :

Protéger les ports des vagues

Une autre application utile est la protection des ports des mouvements ondulatoires de la mer, notamment de la houle. L'objectif : calmer la surface maritime du port. Cette technique existe déjà, par l'utilisation de gros blocs de béton, mais ne répond plus aux attentes écologiques actuelles. Pour y remédier, Kim Pham et son équipe proposent une ceinture de résonnance flottante[1], qui entourera le port à protéger. Elle est composée, pour le moment, de plaques de plexiglas, l'important étant que celles-ci soient étanches. Ce prototype se limite cependant aux expériences en laboratoire, le choix du matériau n'est donc pas encore déterminant. L'idée restant d'aller à terme vers des structures flottantes, légères et résistantes. À l'intérieur de cette ceinture se trouve une petite cavité qui laissera passer les ondes de cette houle. Une fois les ondes emprisonnées dans la ceinture, elles résonneront en son sein jusqu'à leur épuisement.

#3 Les ondes sismiques :

Rediriger les tremblements dans le sol

#4 Les rayons lumineux :

Rendre un objet invisible

Metamaterials

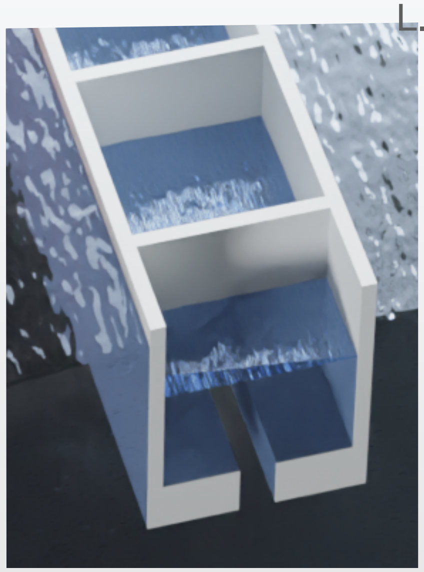
#2 Les ondulations maritimes :

Protéger les ports des vagues

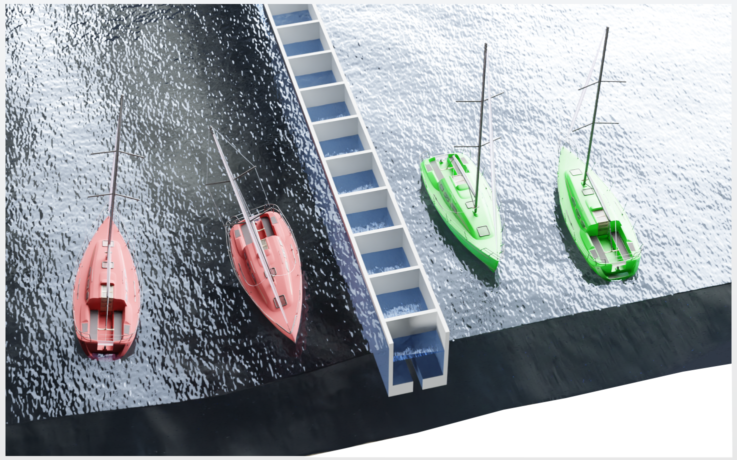
Ceinture de protection (metainterface)

The building block is a resonator

a cavity open to the sea
(a dock for close hole)



Time domain modelling of a
Helmholtz resonator analogue for water waves
L.-P. Euvé, K. Pham, P. Petitjeans, V. Pagneux, A. Maurel
JFM 2021



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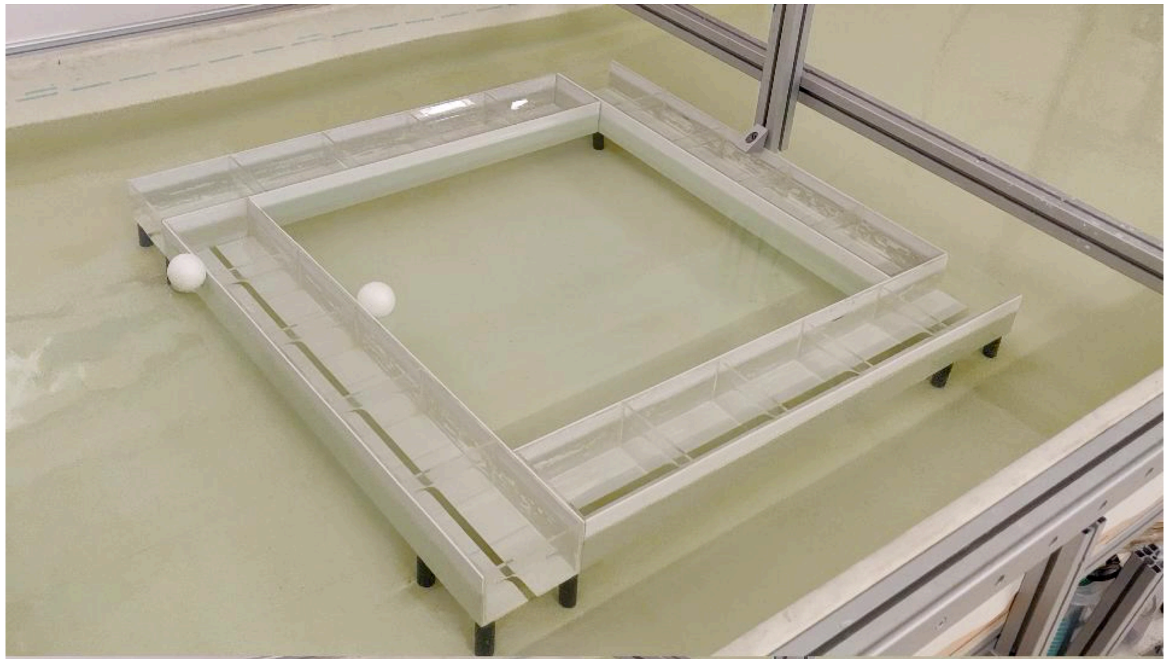
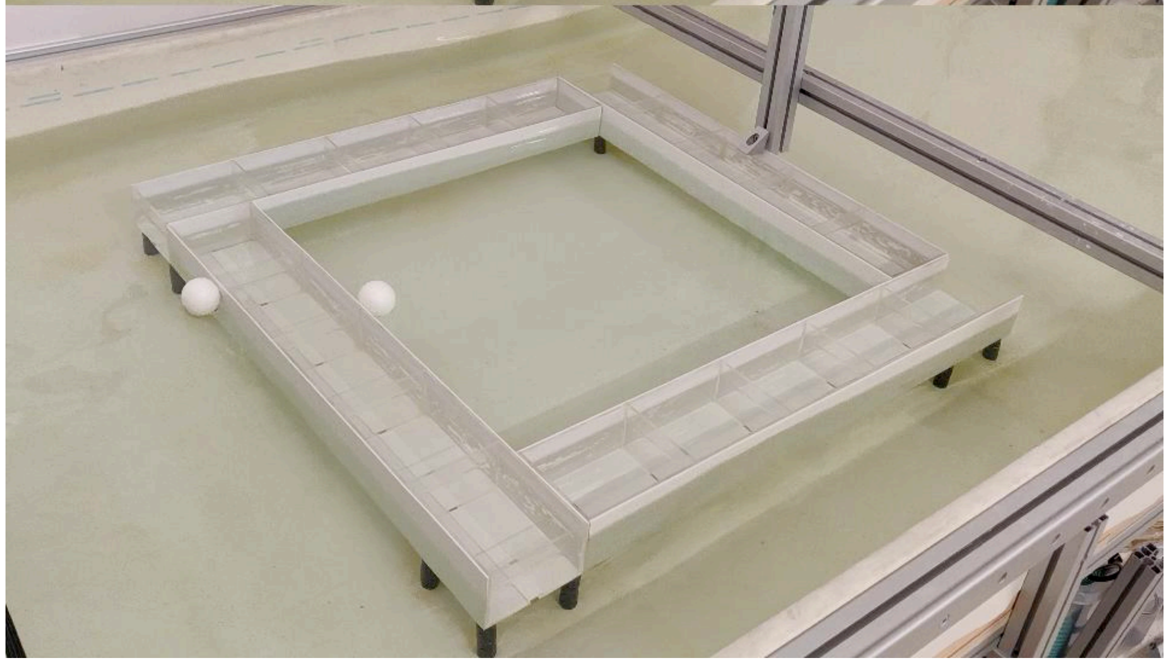
#2 Les ondulations maritimes :

Protéger les ports des vagues

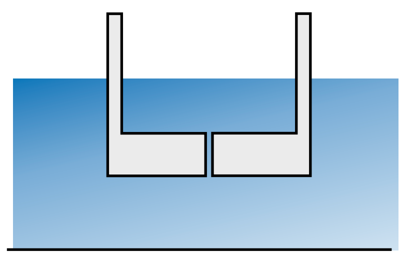
protected



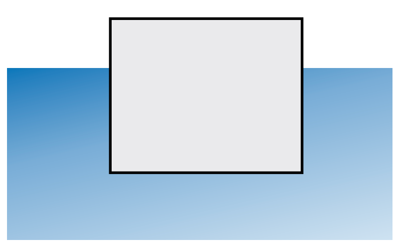
unprotected



Resonators



Docks



Metamaterials

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What is the definition of a metamaterial ?

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The scientific activity (mechanics, physics) on metamaterials started in the 2000s

Reinforced concrete (roughly 1850)

It is a composite material made of concrete (mixture of stones) in which steel is embedded.

Concrete has high resistance in compression but low resistance in bending.
Steel is bend resistant
Concrete and steel act together resulting in a composite material which resists compression but also bending.

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Reinforced concrete (roughly 1850). Why reinforced concrete is not considered as a metamaterial ?

There are no absolute answers to this question.

A possible reason is that an extraordinary property refers to a new physics, which defies the classical ones.

Metamaterials

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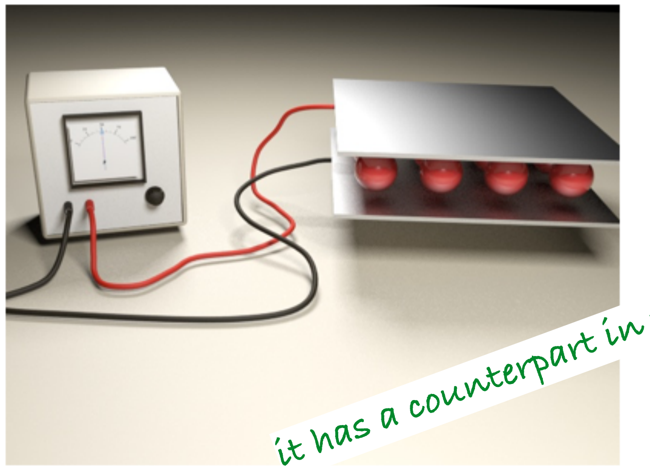
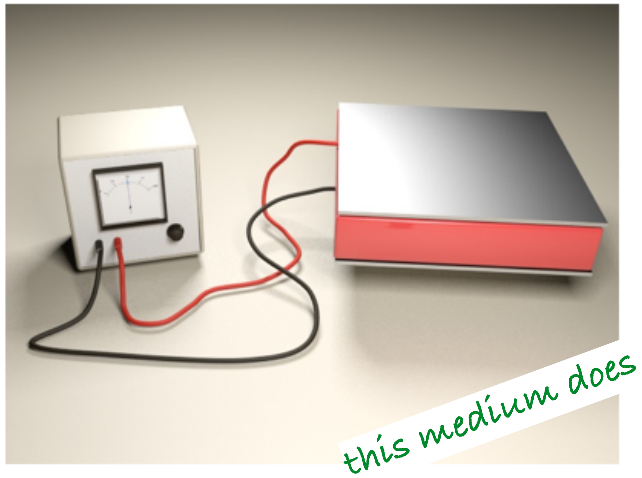
- Homogenization aims to replace a micro-structured medium (inhomogeneous) by an equivalent, homogeneous, one





same response (say voltage)

http://people.ee.duke.edu/~drsmith/metamaterials/metamaterials_homogenization.htm

- Why doing so ?

because it is much simpler theoretically to optimise the properties of a homogeneous medium than those of an inhomogeneous medium.

- And then ?

Once the equivalent homogenous medium has been optimised (with respect to the desired extraordinary property), you can to come back to the actual microstructure able to produce in practice the extraordinary property.

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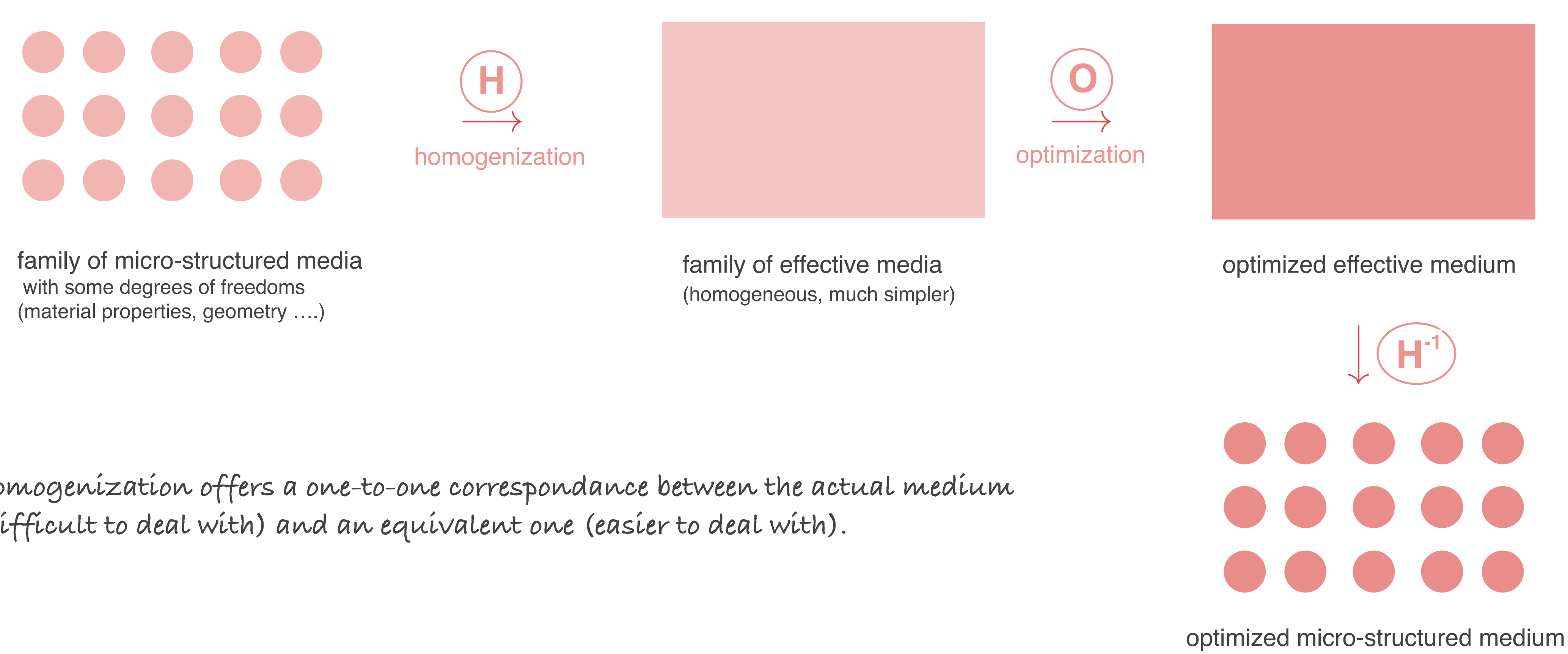

Homogenization offers a one-to-one correspondance between the actual medium (difficult to deal with) and an equivalent one (easier to deal with).

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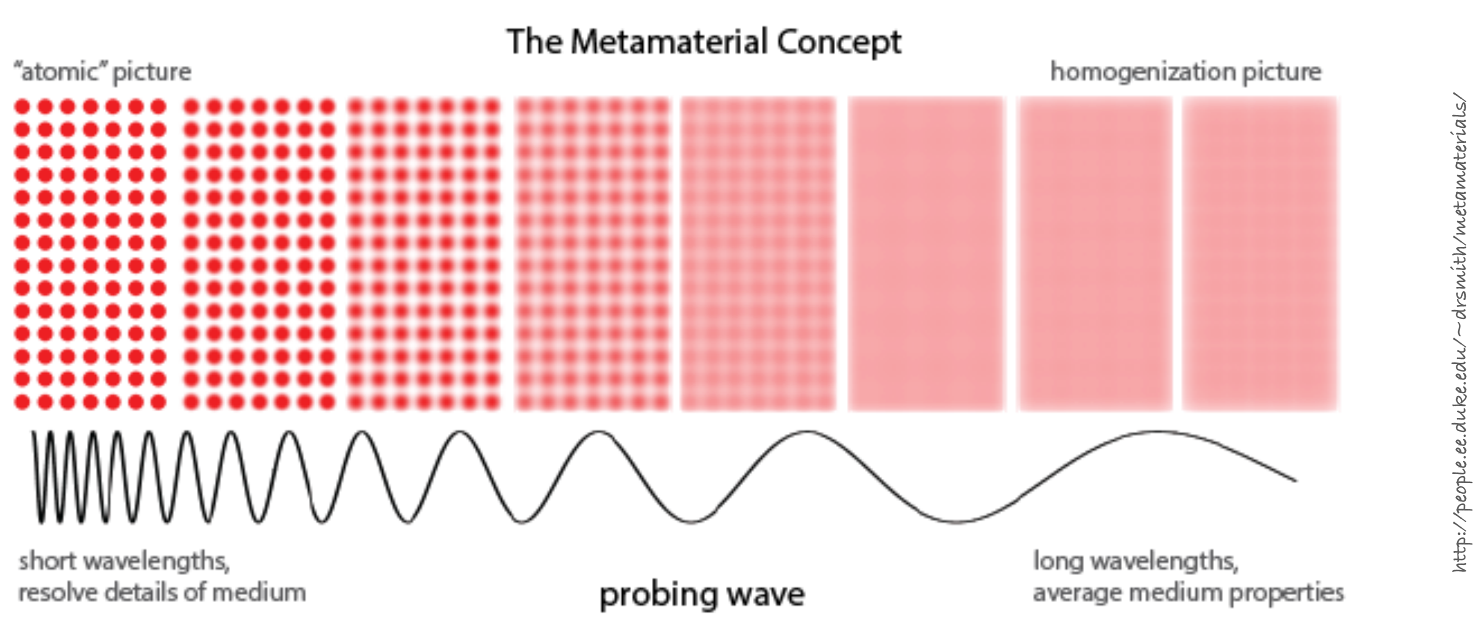
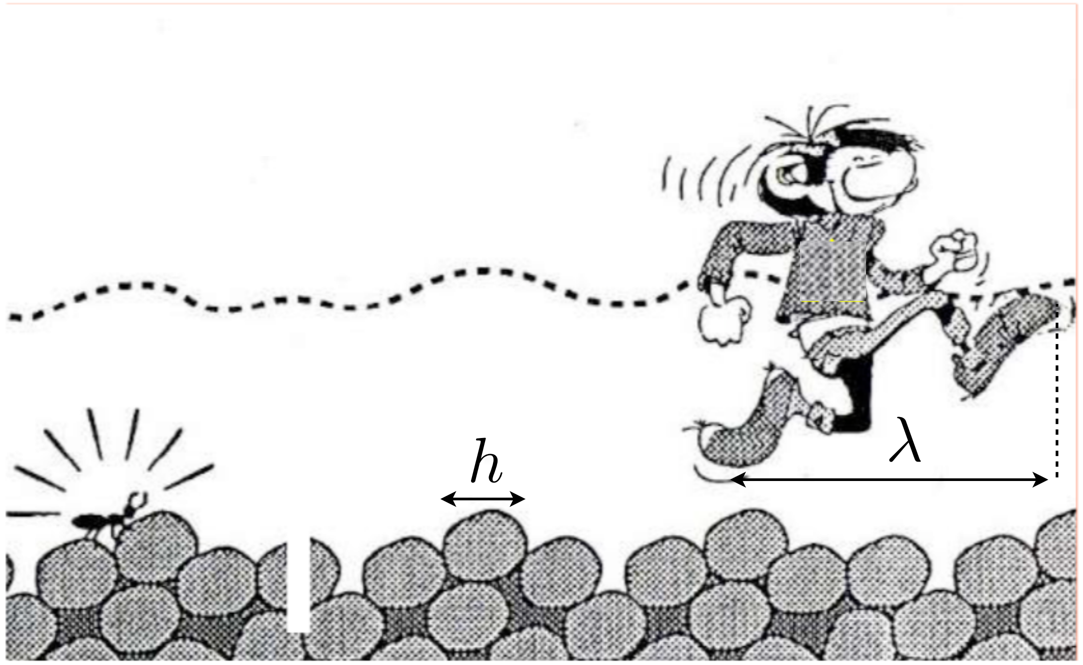
- Can we homogenise any structure ? NO

In a dynamical context, homogenization makes sense if the wavelength is (much) larger than the spacing (**subwavelength micro-structures**)

Classical homogenization concerns **periodic micro-structures**

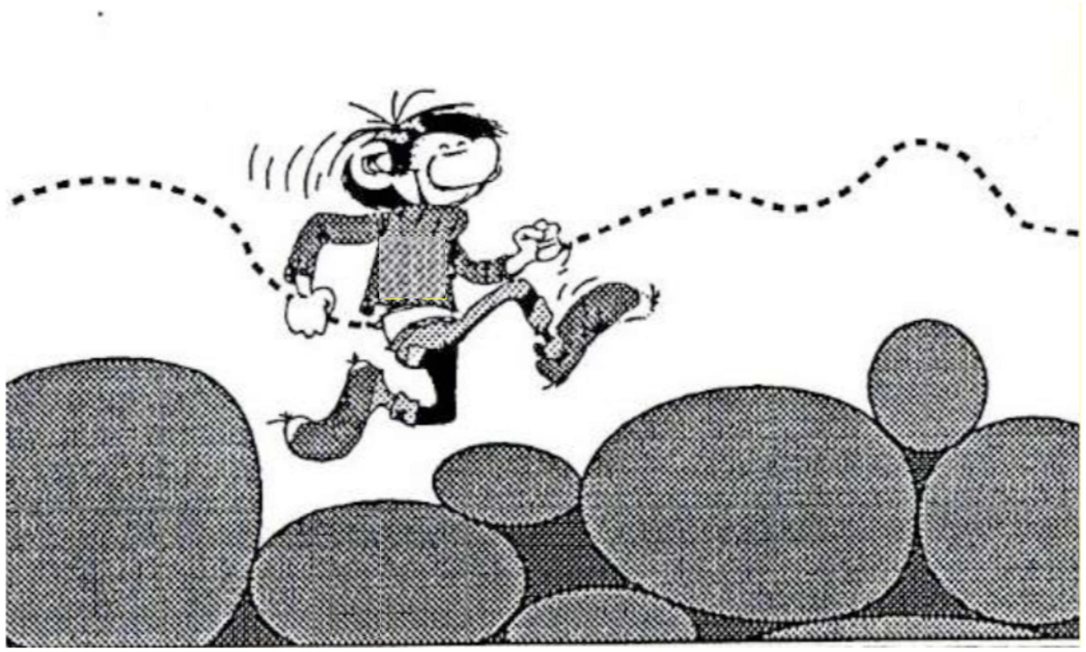


Metamaterials



$h \ll \lambda$

Effective speed



$h \sim \lambda$

[Boutin et Auriault]

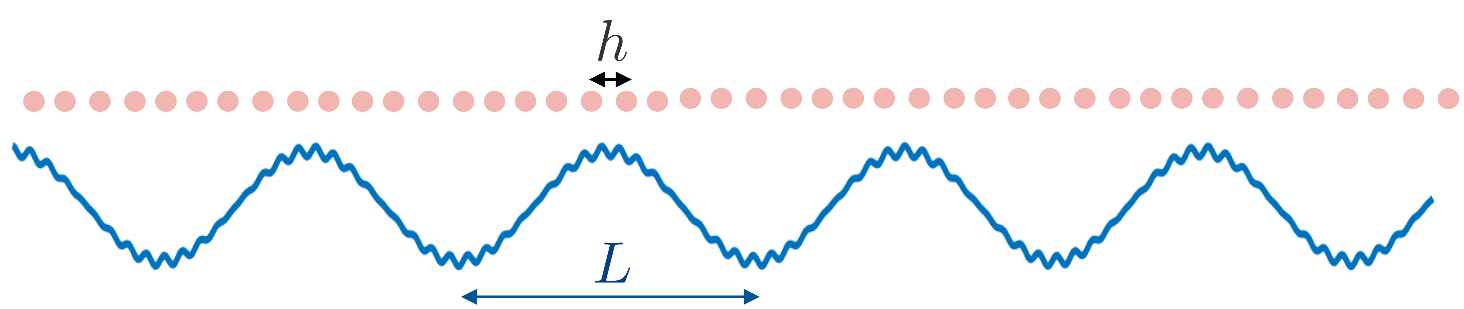
Metamaterials

- Asymptotic homogenization of a microstructure in its bulk

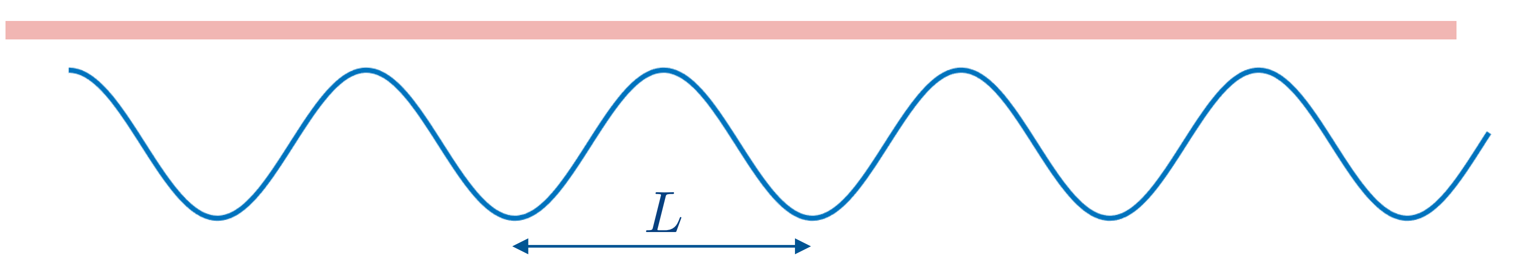
« separation of scales »

homogenization aims to capture the effect of the small scale in an « averaged sense »

the actual problem



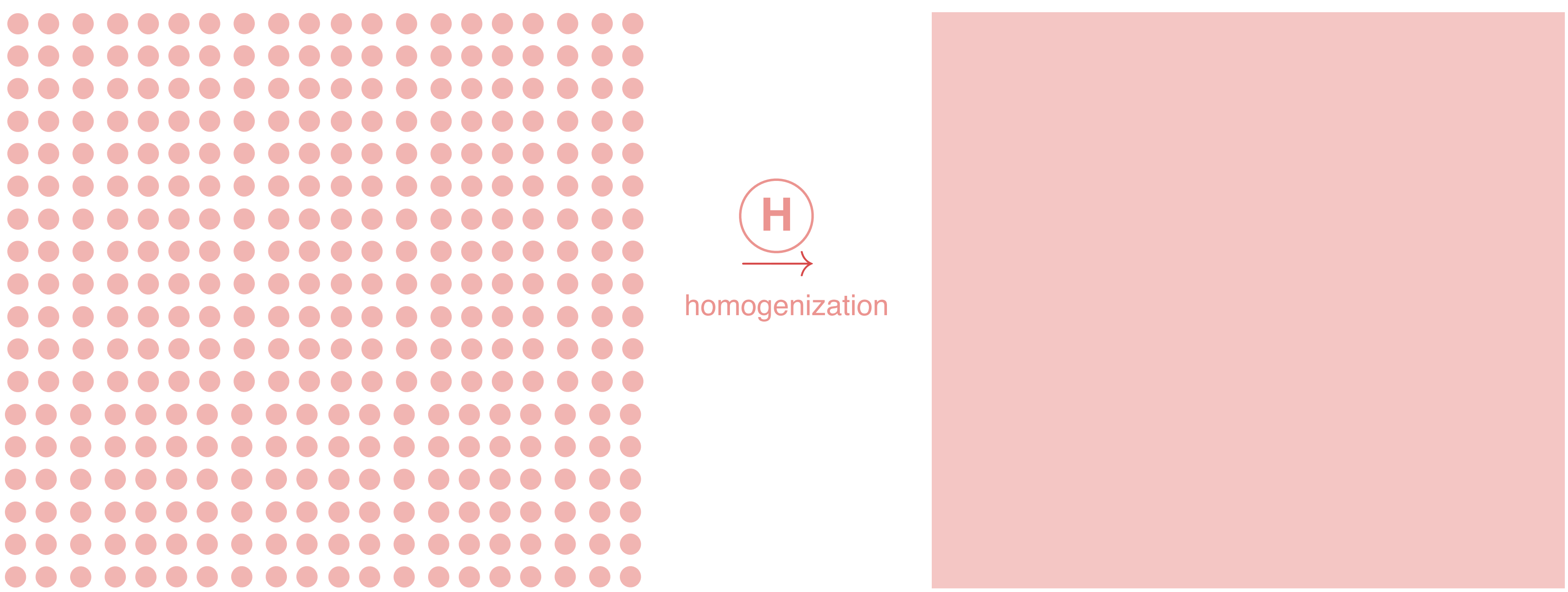
the homogenized problem



means some averaging process during homogenization

Metamaterials

- Asymptotic homogenization of a microstructure in its bulk



Metamaterials

- Asymptotic homogenization of a microstructure in its bulk

We shall work with the simplest wave equation :

wave equation written in terms of a scalar field $p(\mathbf{x}, t)$ a vector field $\mathbf{u}(\mathbf{x}, t)$
with material parameters $a(\mathbf{x})$ and $b(\mathbf{x})$

$$\frac{\partial \mathbf{u}}{\partial t} = -a\,\nabla p \qquad \mathrm{div}\mathbf{u} + b\,\frac{\partial p}{\partial t} = 0 \qquad \longrightarrow \qquad \frac{\partial^2 p}{\partial t^2} - \frac{a}{b}\,\Delta p = \mathbf{0}$$

continuity of p and of $\mathbf{u} \cdot \mathbf{n}$ at the interfaces

This wave equation applies in many contexts of waves

	scalar field ***p***	**vector field** ***u***	**material parameter** ***a***	**material parameter** ***b***
acoustics (full 3d)	pressure *p*	velocity ***u***	inverse of mass density ρ	inverse of bulk modulus $B = \rho c^2$
electromagnetism (2d polarized)	out-of-plane magnetic field *H*	auxiliary field linked to the in-plane electric field ***E***	inverse of permittivity ε	permeability of vacuum μ_0
elastodynamics (2d)	out-of-plane displacement *u*	in-plane vector stress σ	shear modulus μ	mass density ρ

Metamaterials

Asymptotic homogenization

We have seen that :

a microstructured medium can be replaced by an effective homogeneous medium, in general anisotropic

We can also show that:

the maximum anisotropy is reached with a stratified medium

the effective problem in the time domain is associated to a positive energy

Metamaterials

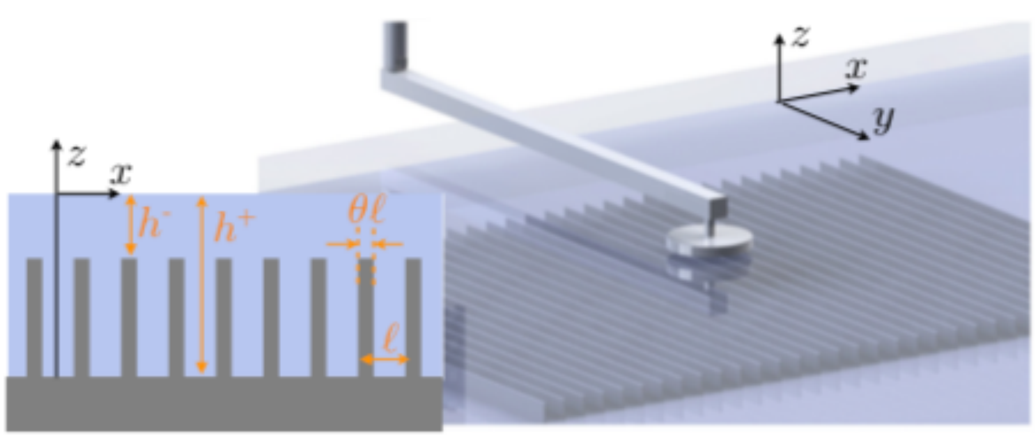
Asymptotic homogenization

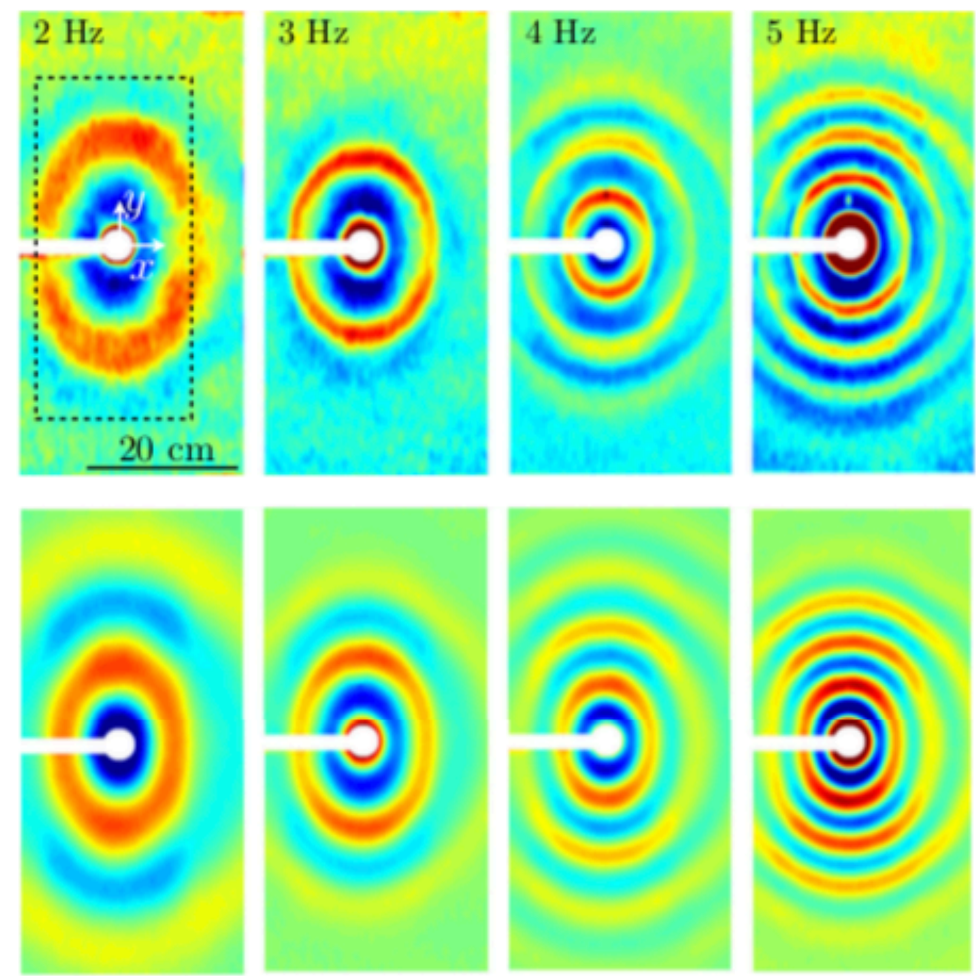
efficiency of the effective problem

PHYSICAL REVIEW B **96**, 134310 (2017)

Revisiting the anisotropy of metamaterials for water waves

A. Maurel, J.-J. Marigo, P. Cobelli, P. Petitjeans, and V. Pagneux



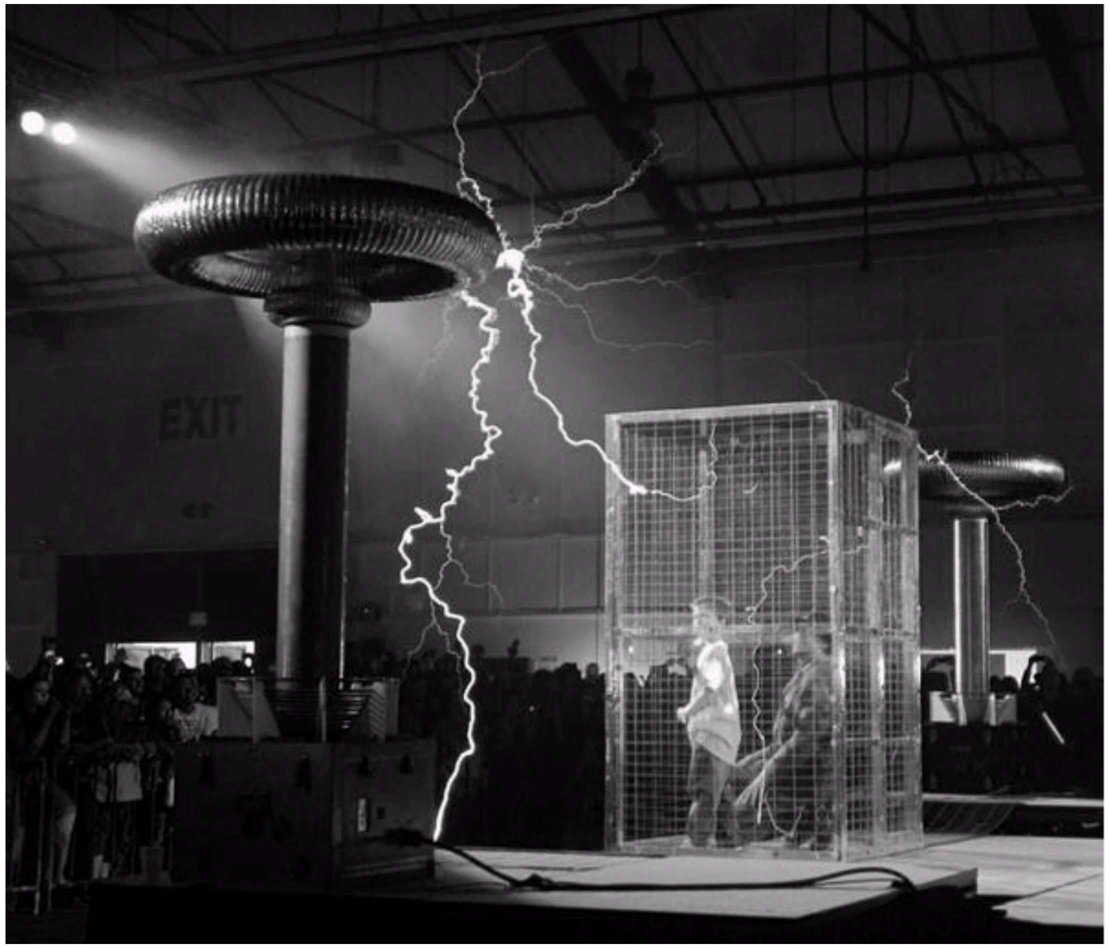


Anisotropy of stratified media in the context of water waves (the swell)

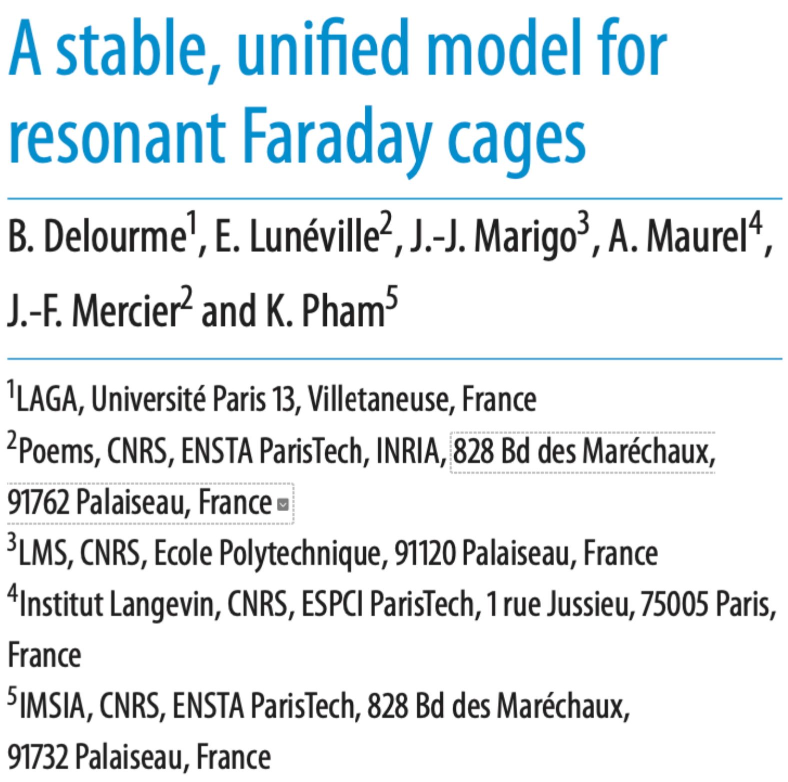
Metamaterials

Asymptotic homogenization

efficiency of the effective problem
stability of the numerics (effective problem)

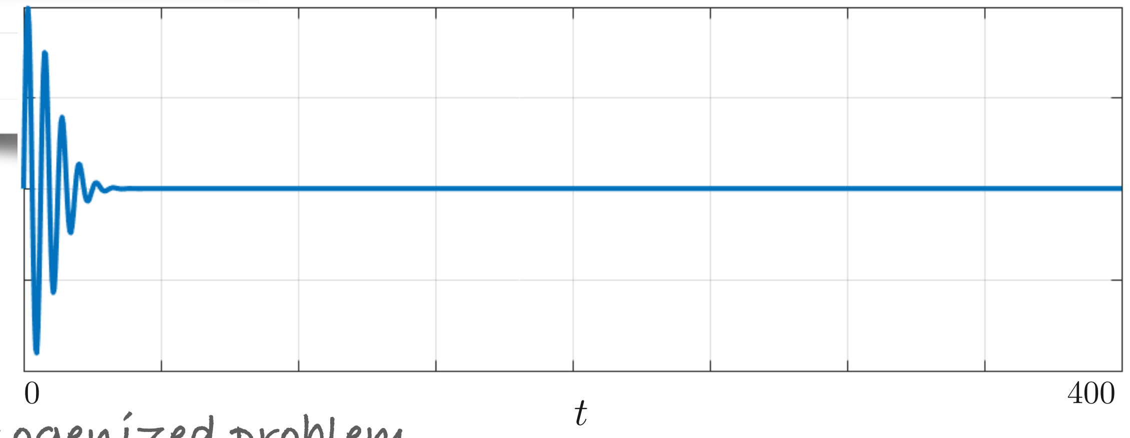


Faraday cage



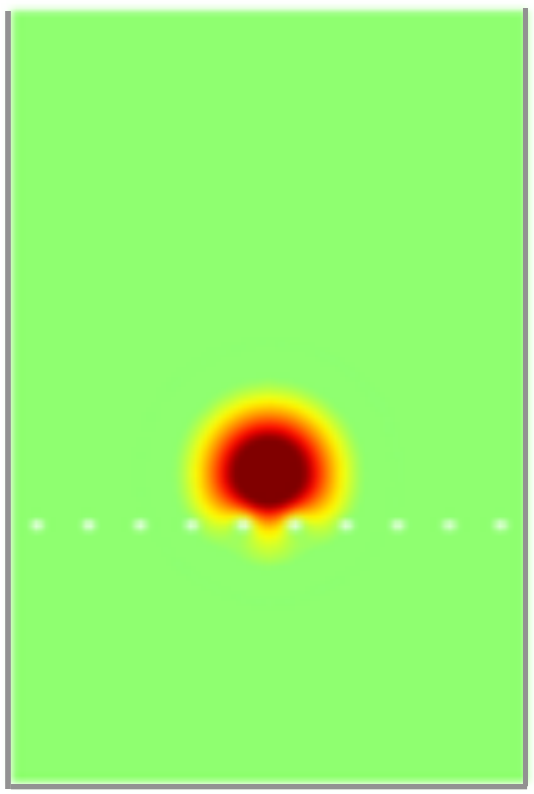
Metamaterials

Asymptotic homogenization

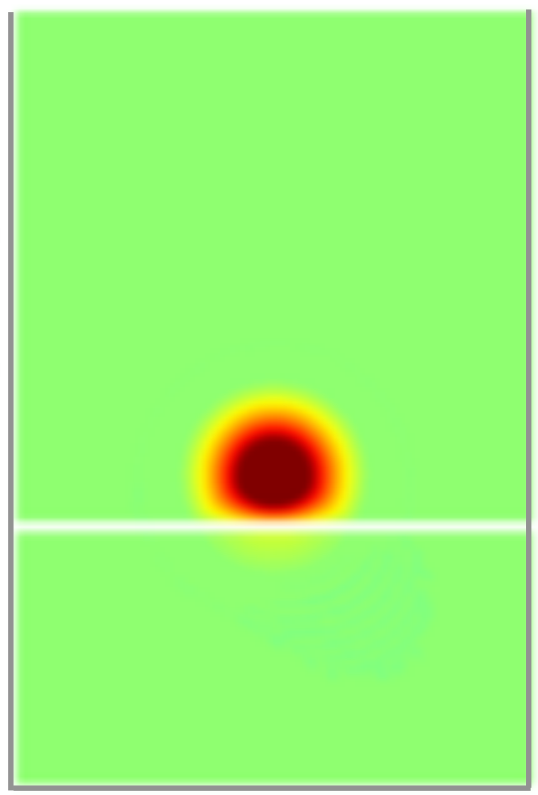


efficiency of the effective problem

real problem



homogenized problem



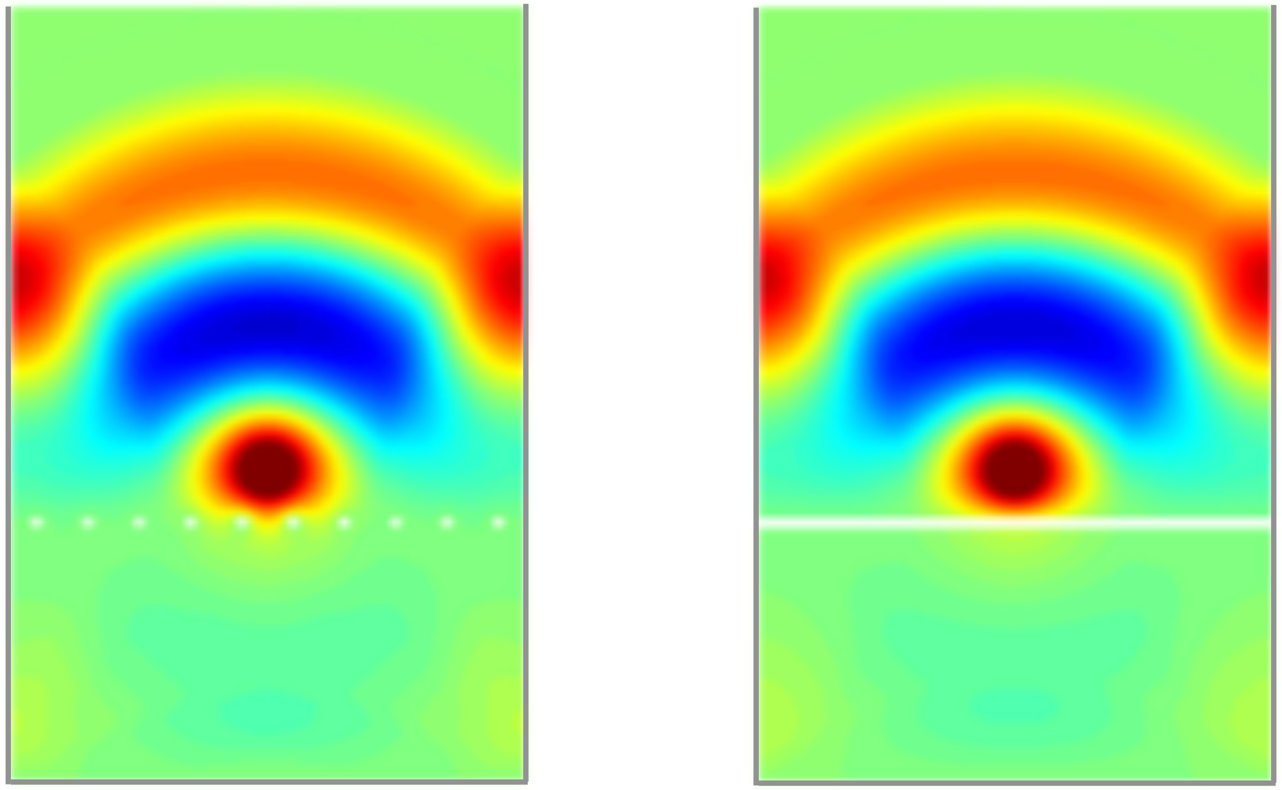
Faraday cage

Metamaterials

Asymptotic homogenization

efficiency of the effective problem

real problem homogenized problem



Faraday cage

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Asymptotic homogenization

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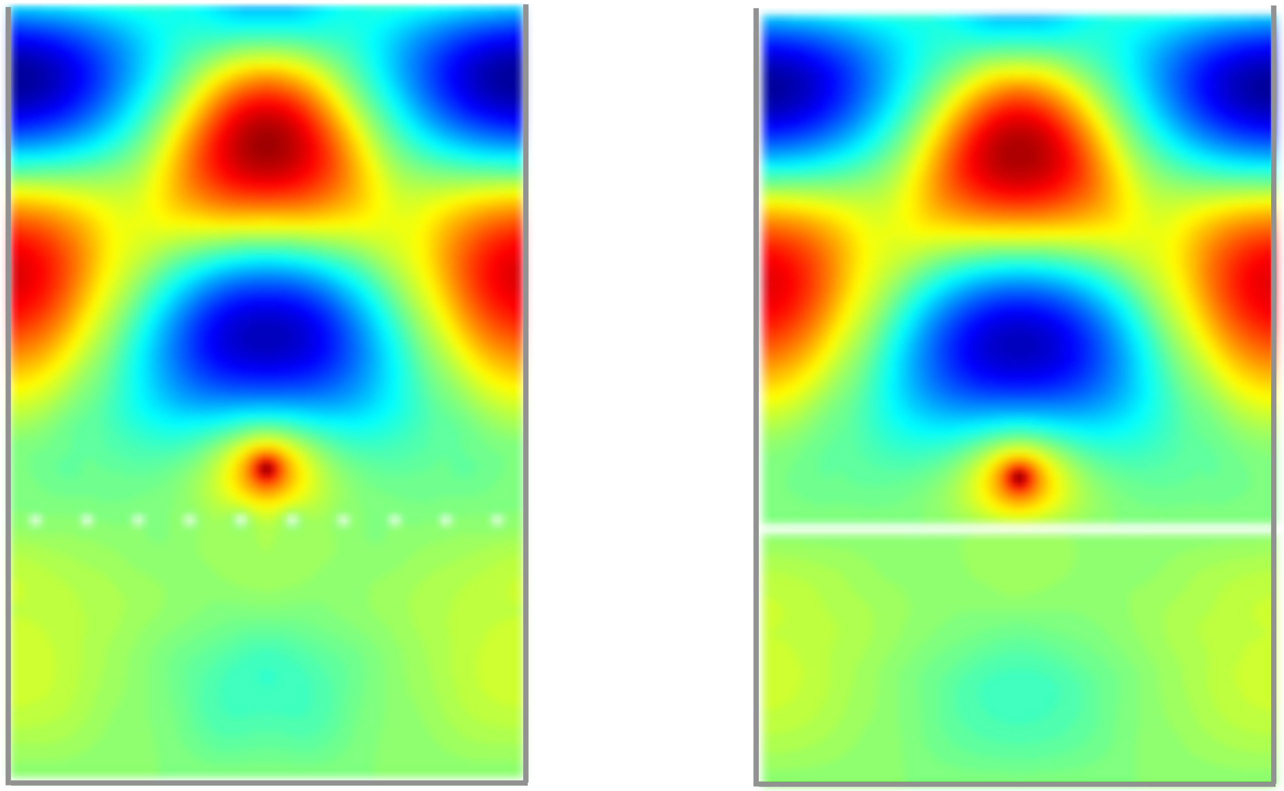

Faraday cage

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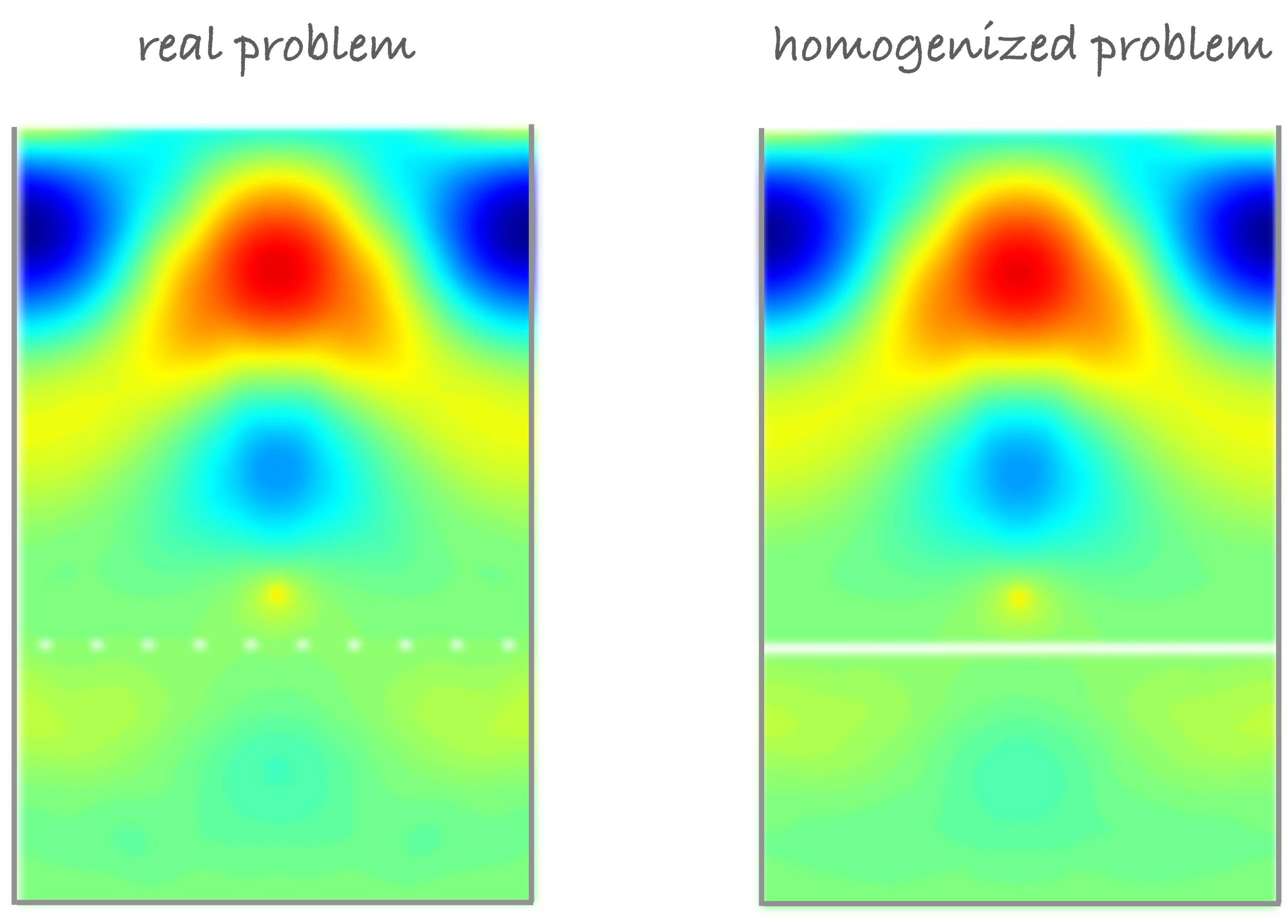
Asymptotic homogenization

efficiency of the effective problem



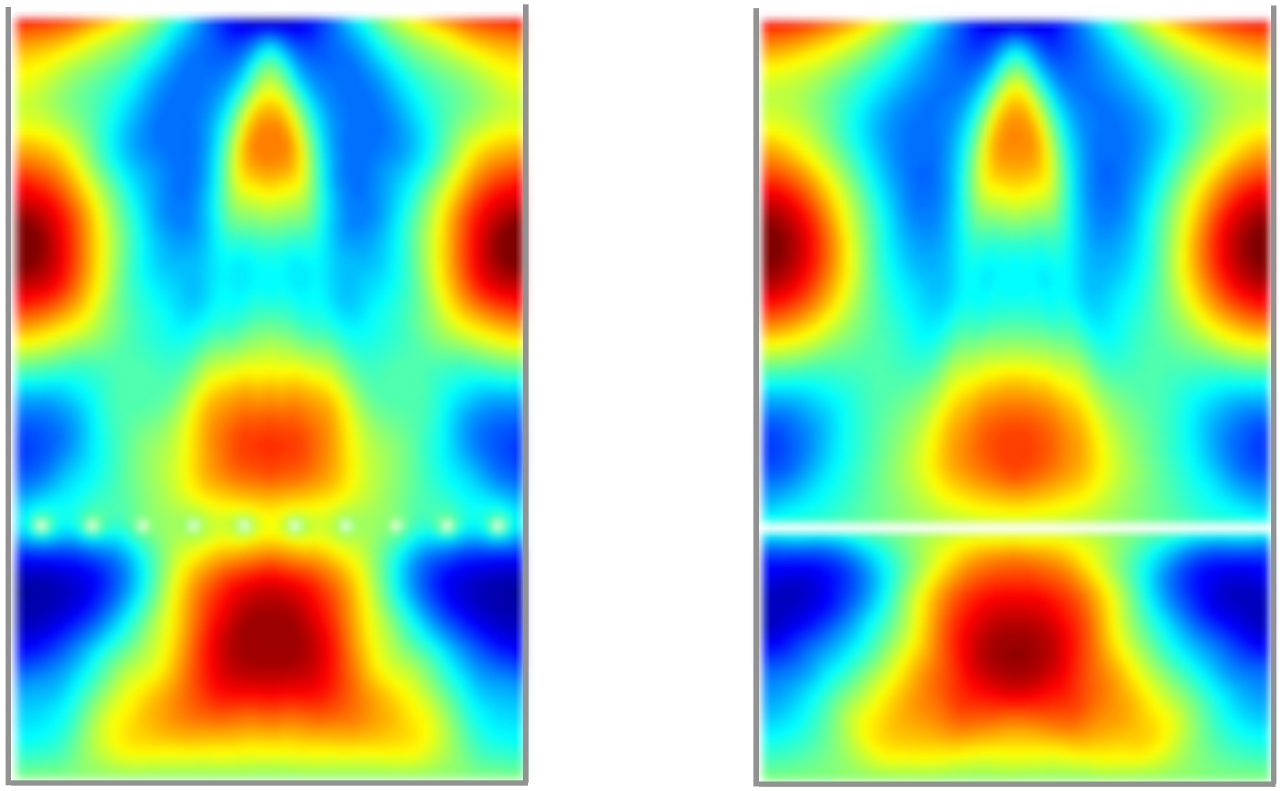
Faraday cage

Metamaterials

Asymptotic homogenization

efficiency of the effective problem

real problem | homogenized problem



Faraday cage

Metamaterials

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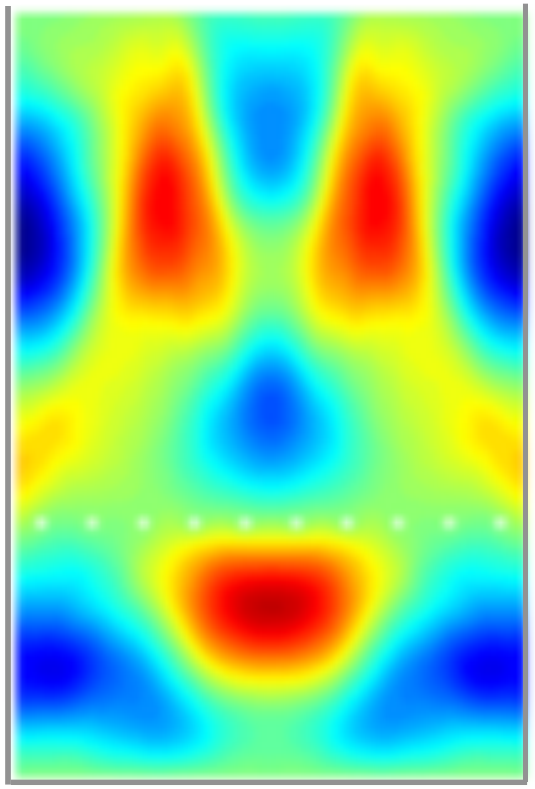
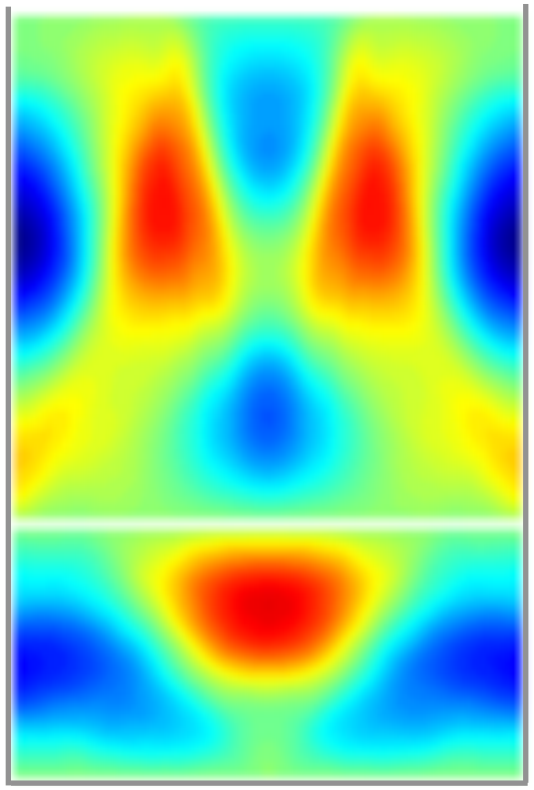


homogenized problem

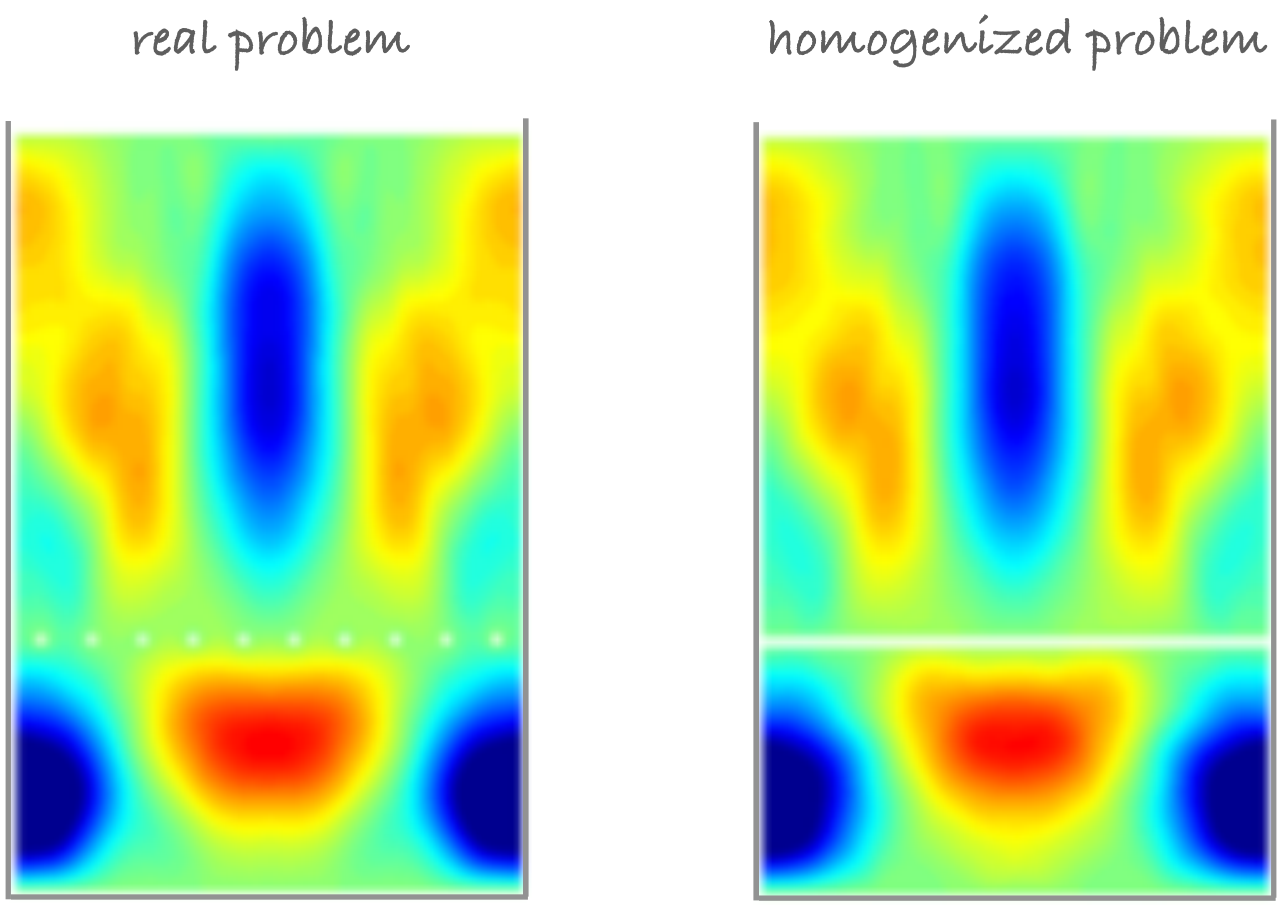


Faraday cage

Metamaterials

Asymptotic homogenization

efficiency of the effective problem



Faraday cage

Metamaterials

Asymptotic homogenization

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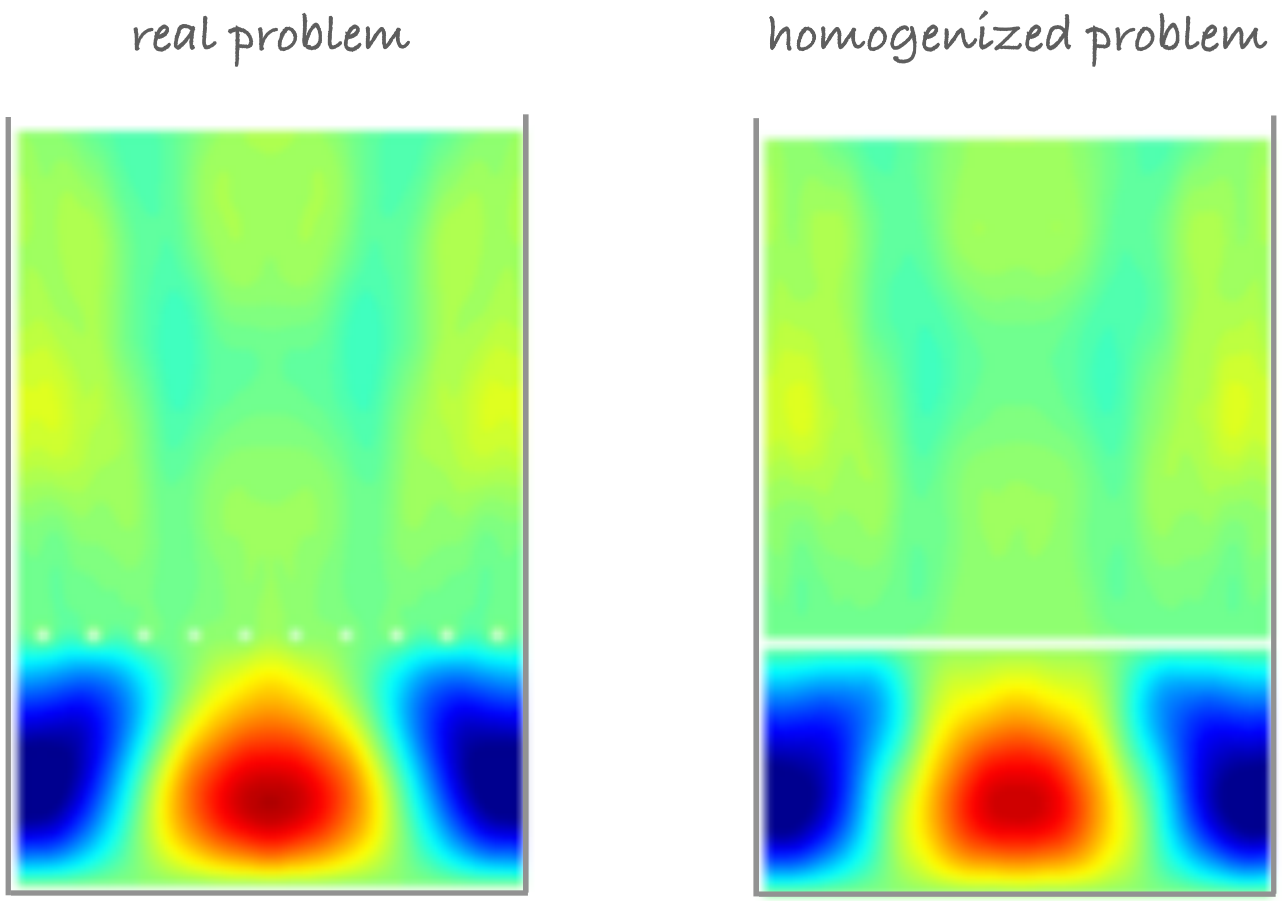

Faraday cage

Metamaterials

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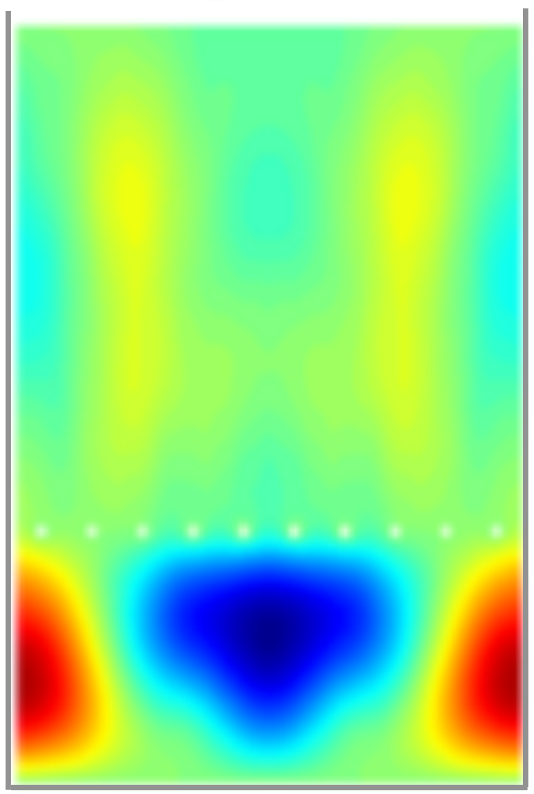
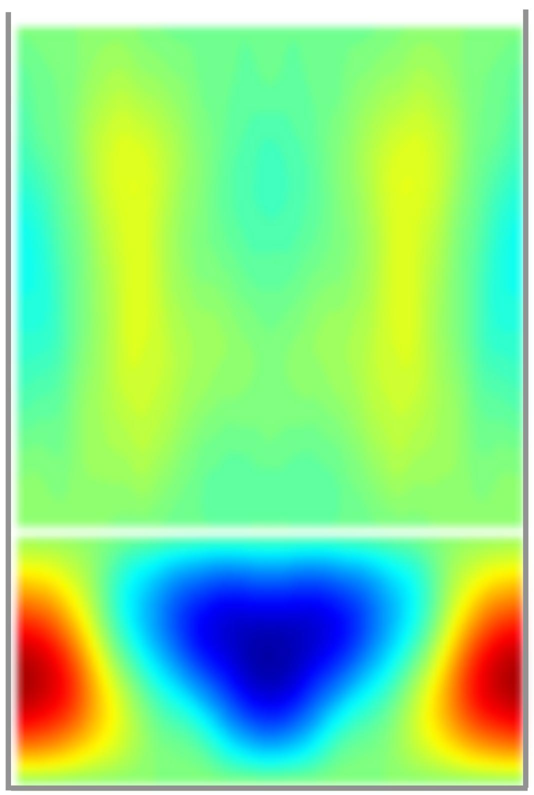


homogenized problem

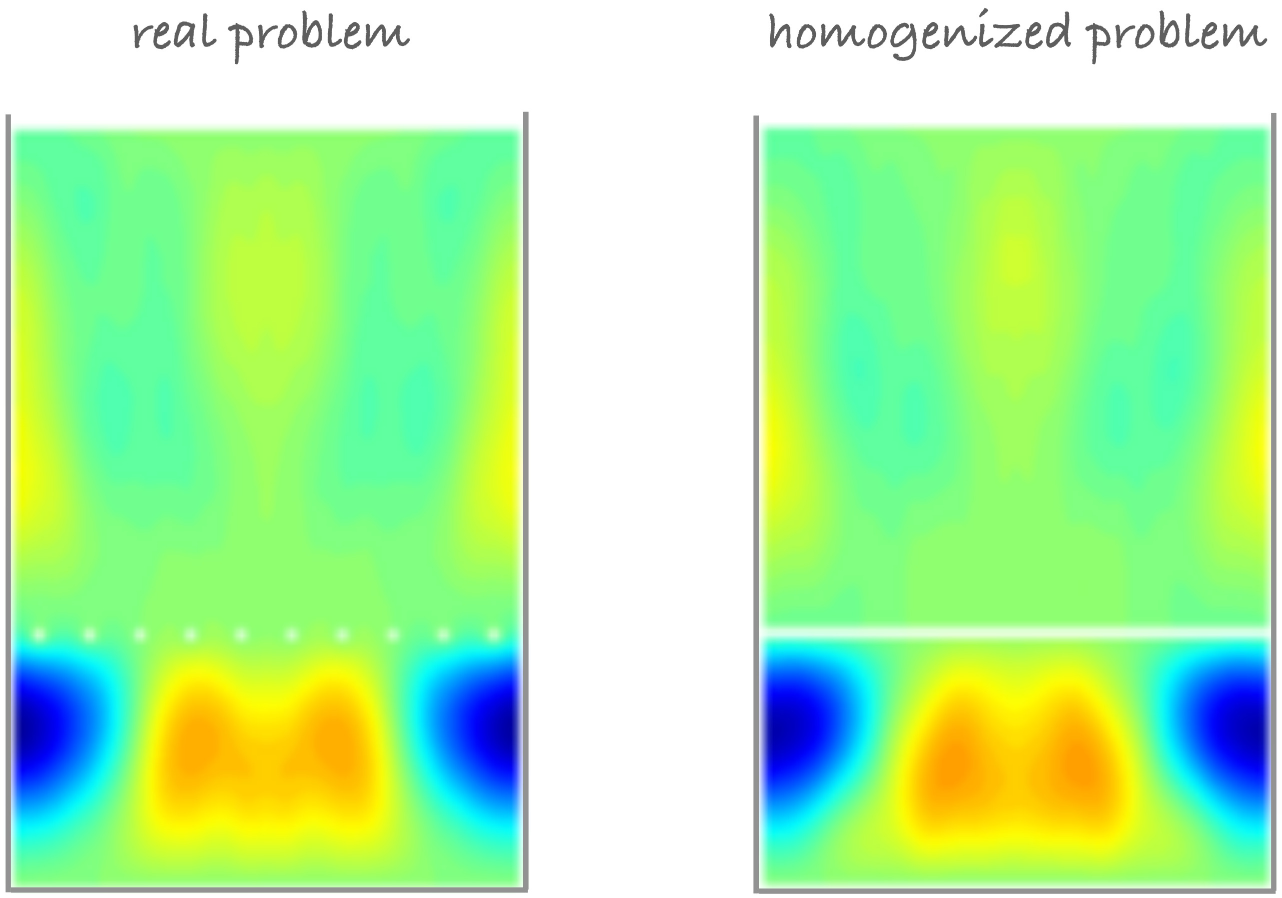


Faraday cage

Metamaterials

Asymptotic homogenization

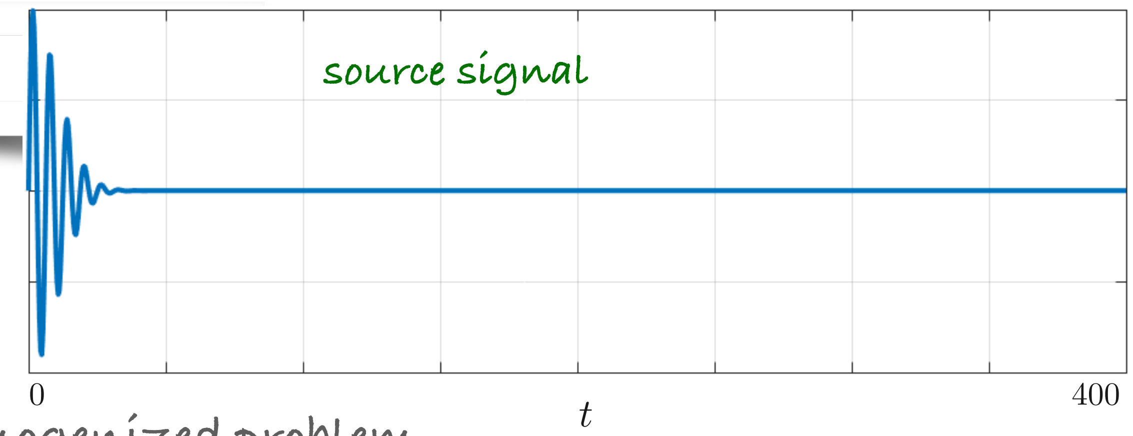
efficiency of the effective problem



Faraday cage

Metamaterials

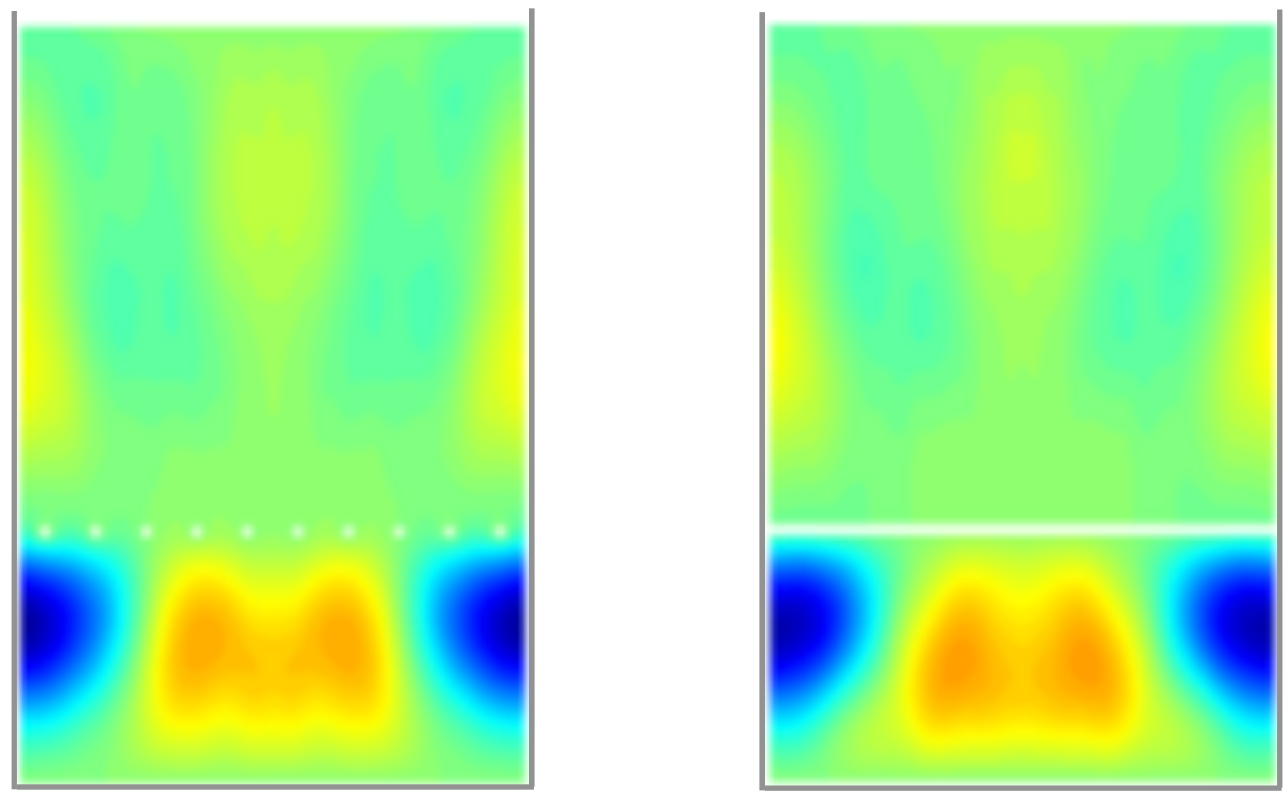
Asymptotic homogenization



efficiency of the effective problem

real problem

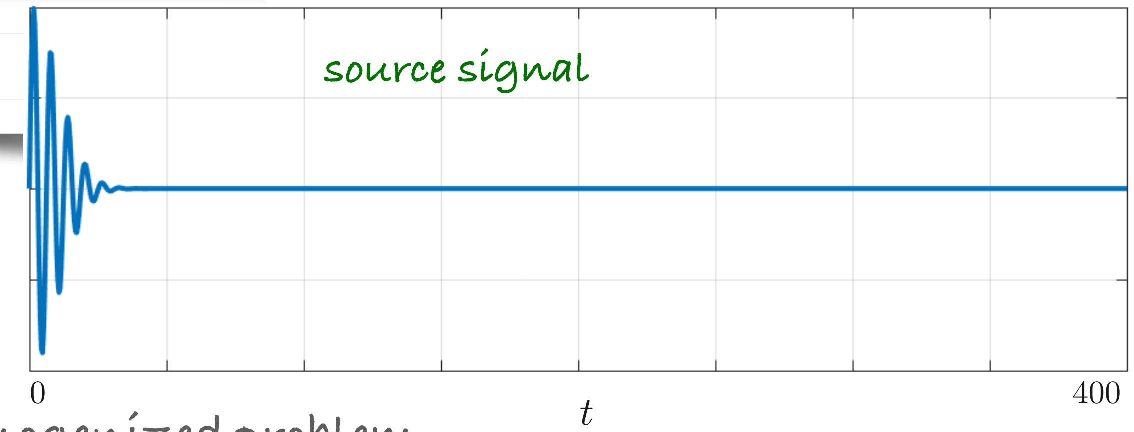
homogenized problem



Faraday cage

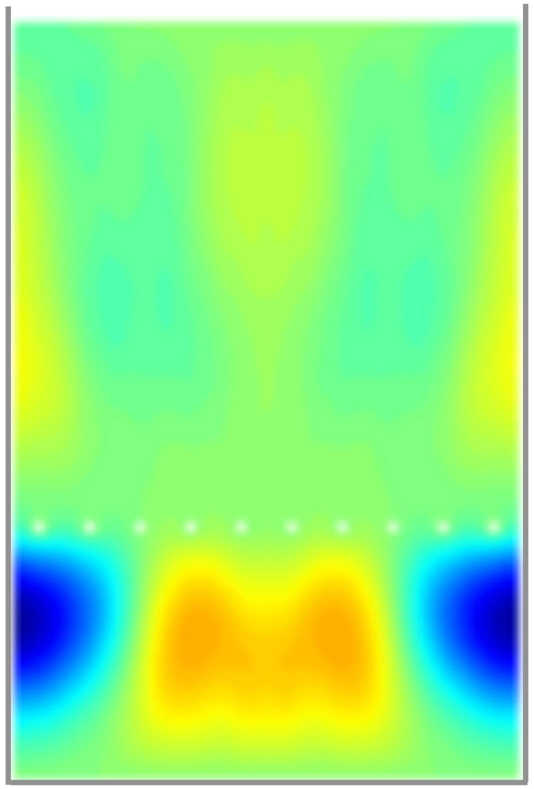
Metamaterials

Asymptotic homogenization

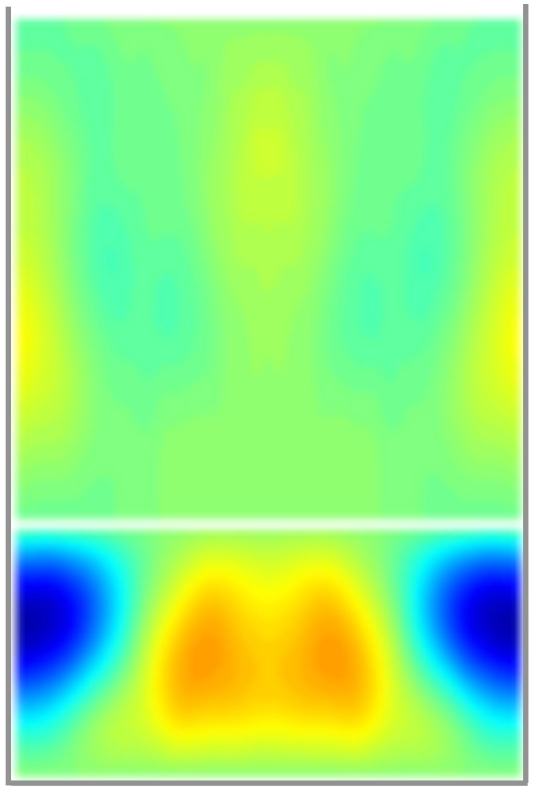


efficiency of the effective problem

real problem



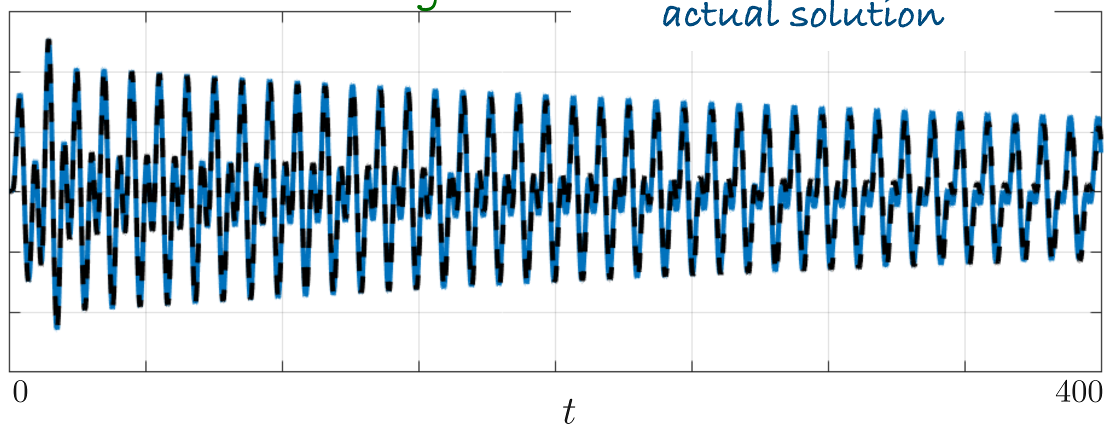
homogenized problem



homogenized solution

signal in

actual solution



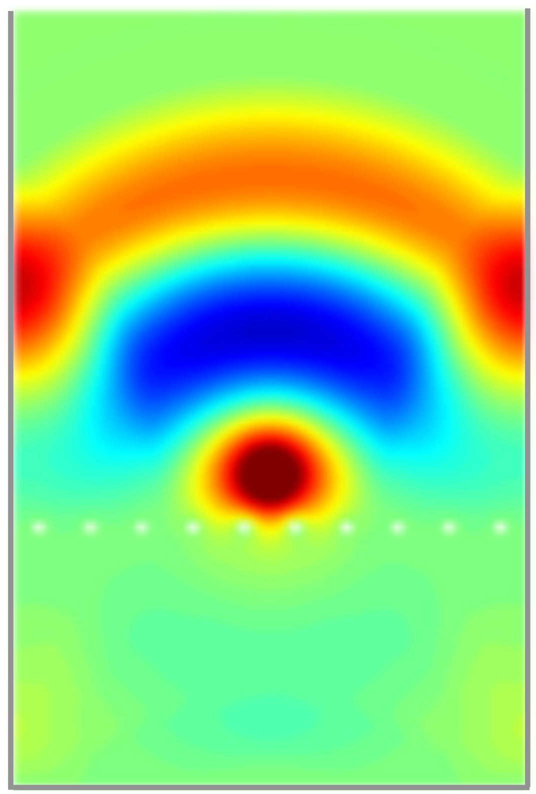
Metamaterials

Asymptotic homogenization

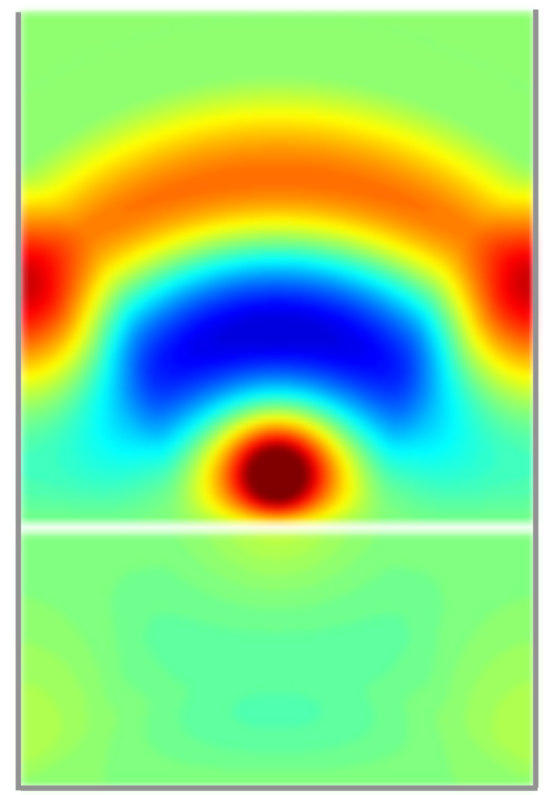
efficiency of the effective problem

here a stable formulation of the effective problem

real problem



homogenized problem

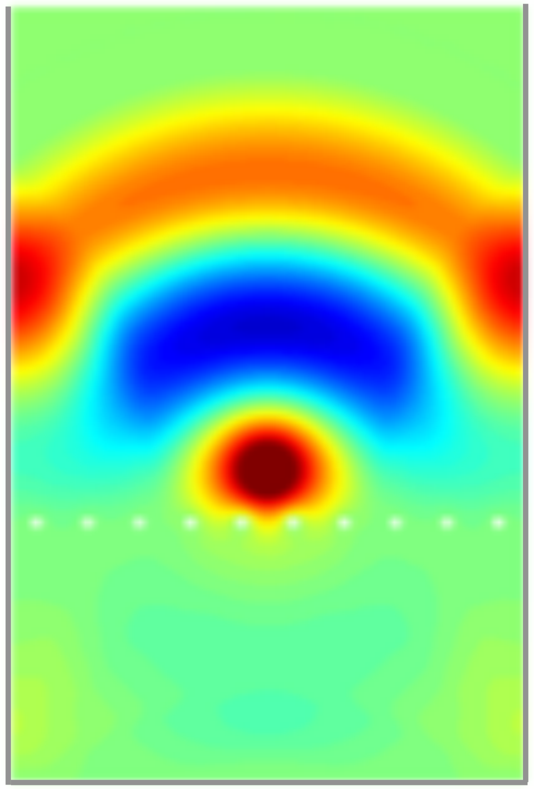


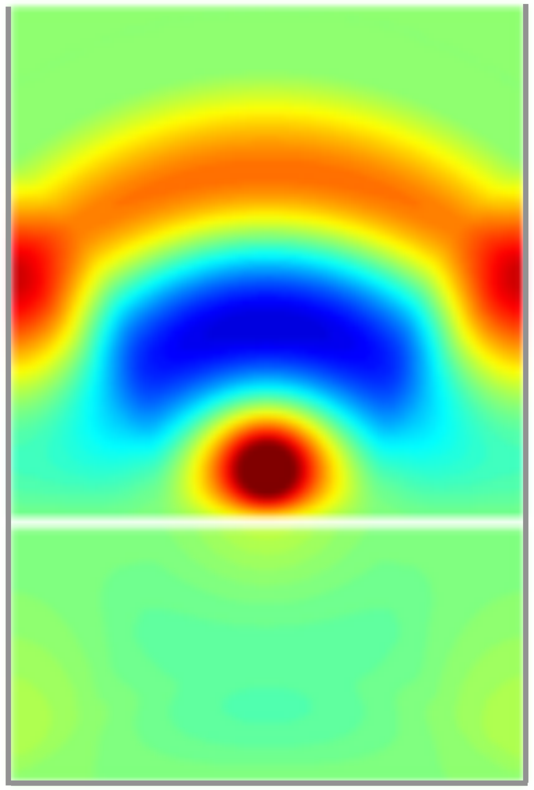
Metamaterials

Asymptotic homogenization

efficiency of the effective problem

stability of the numerics (effective problem)





stable formulation (associated to a positive energy)

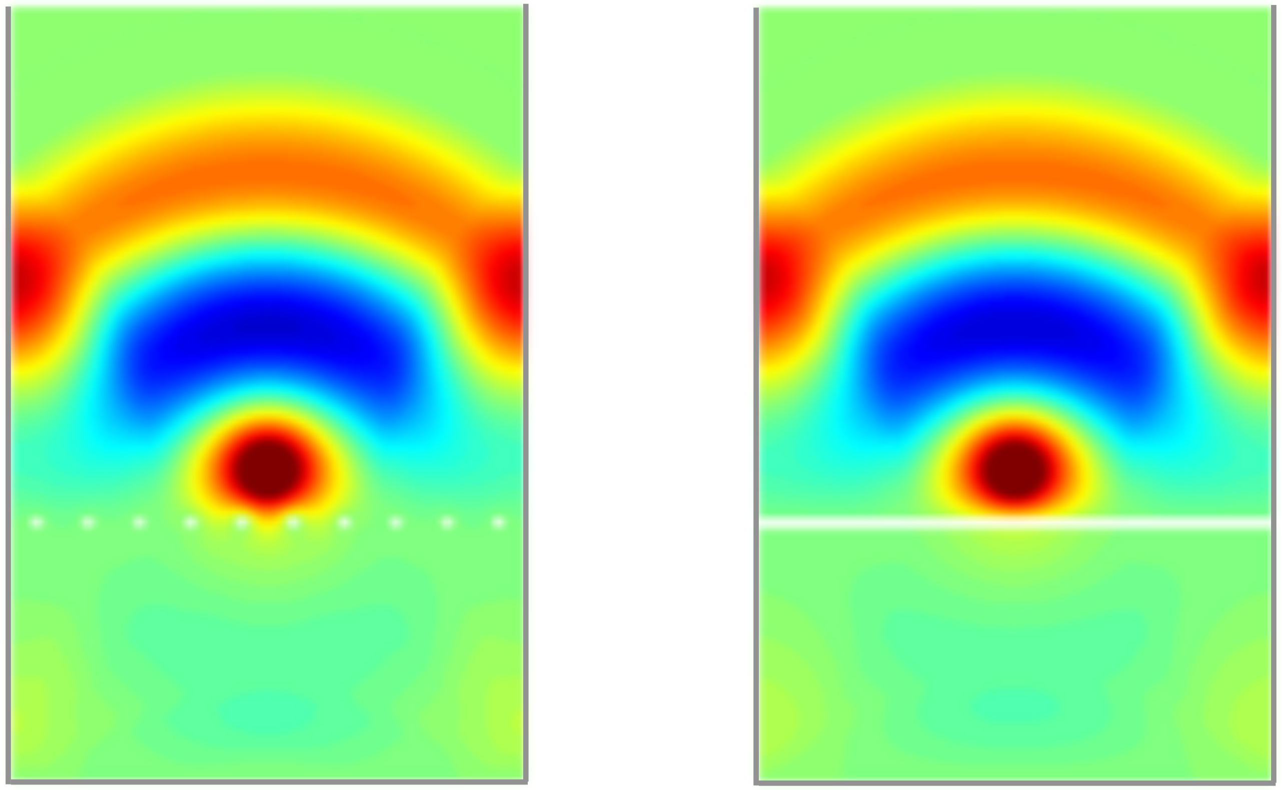
Metamaterials

Asymptotic homogenization

efficiency of the effective problem
stability of the numerics (effective problem)

real problem

homogenized problem



stable formulation (associated to a positive energy)

↓

unstable formulation (associated to a negative energy)

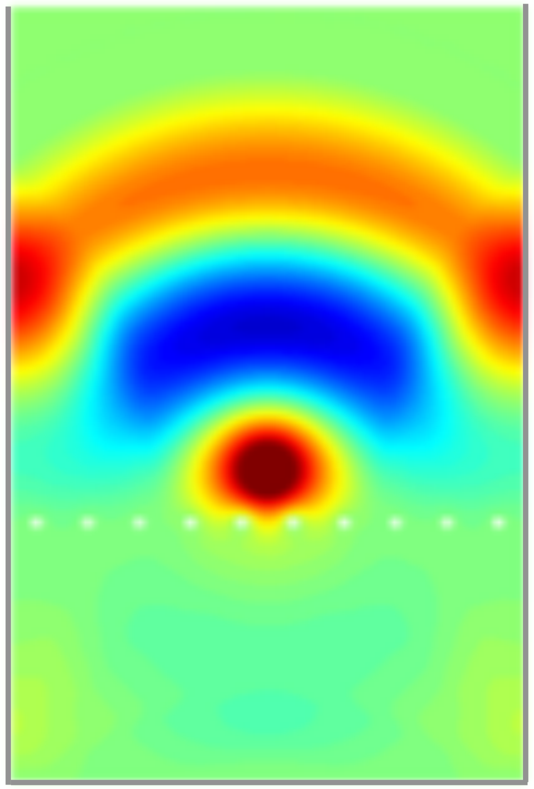
Metamaterials

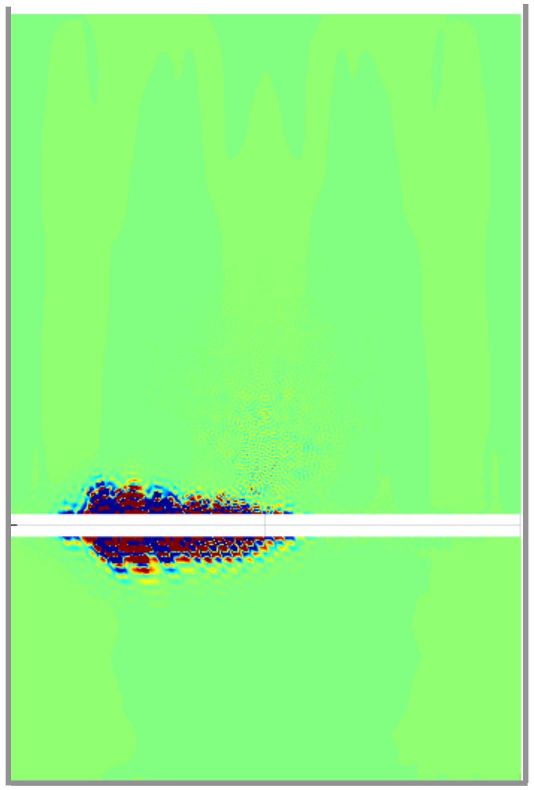
Asymptotic homogenization

efficiency of the effective problem
stability of the numerics (effective problem)

real problem

homogenized problem





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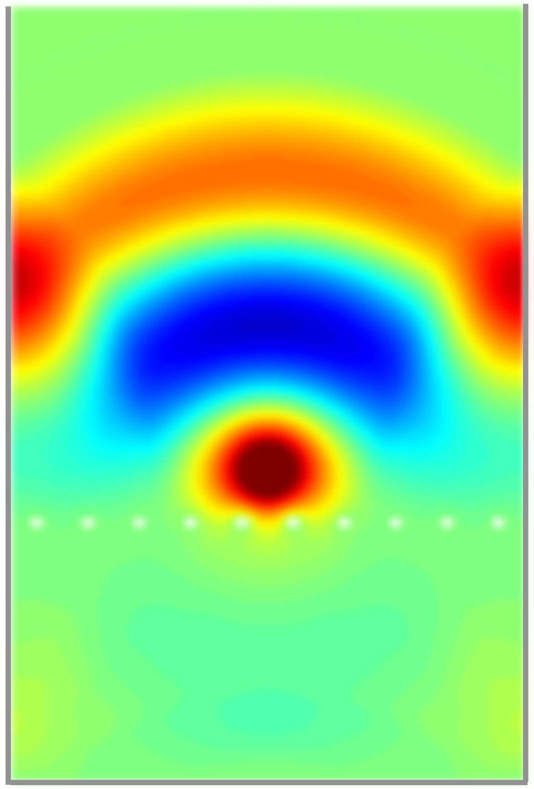

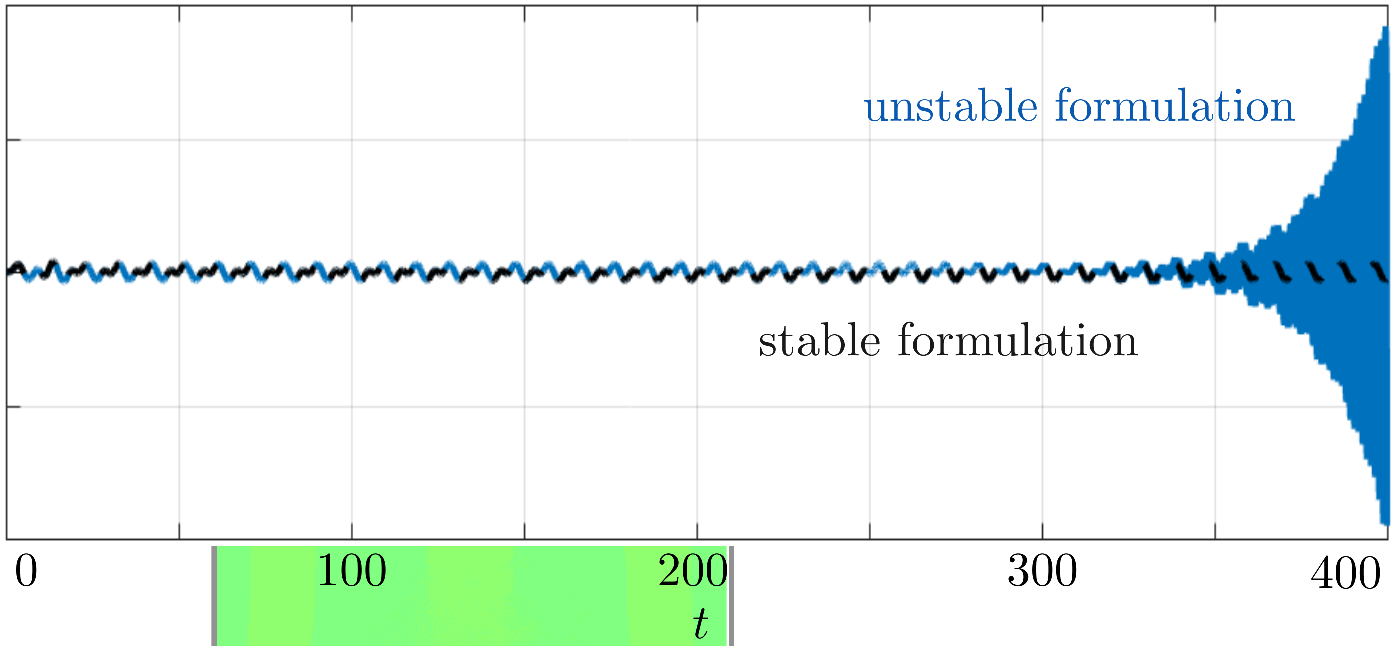

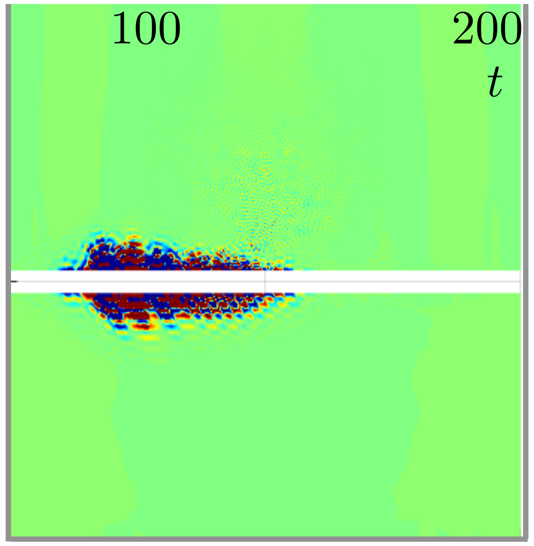

stable formulation (associated to a positive energy)

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unstable formulation (associated to a negative energy)

Metamaterials

Au delà de l'ordre dominant
L'anomalie de Segev

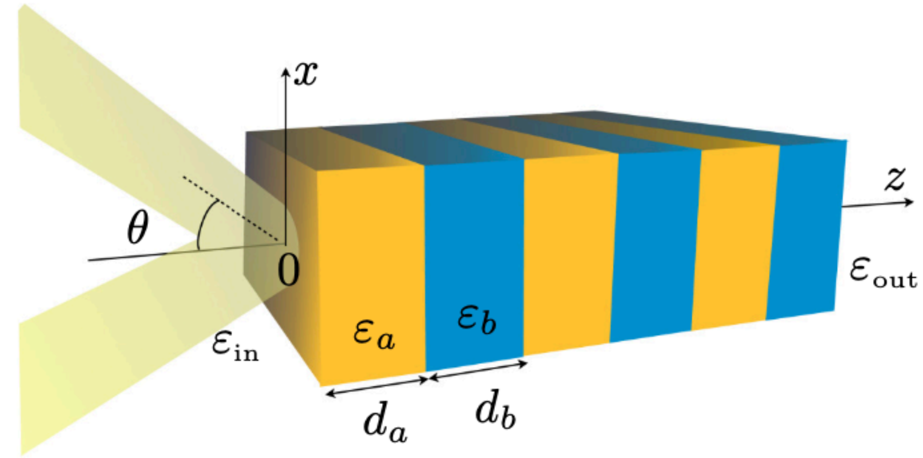


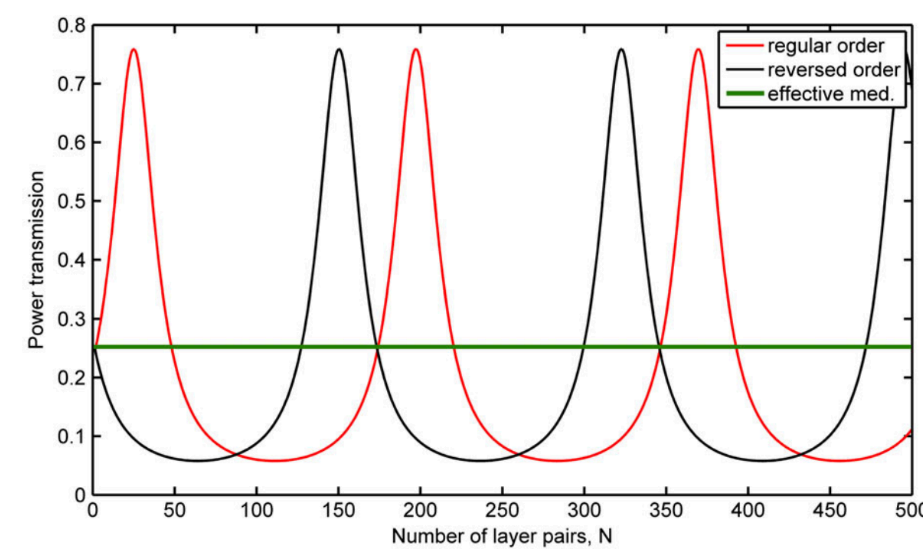
FIG. 1. Multilayer structure illuminated by a plane wave near the critical angle of total reflection ($d = d_a + d_b$ is the period).

PRL **113,** 243901 (2014) PHYSICAL REVIEW LETTERS week ending 12 DECEMBER 2014

Subwavelength Multilayer Dielectrics: Ultrasensitive Transmission and Breakdown of Effective-Medium Theory

Hanan Herzig Sheinfux, Ido Kaminer, Yonatan Plotnik, Guy Bartal, and Mordechai Segev
Technion–Israel Institute of Technology, Haifa 32000, Israel

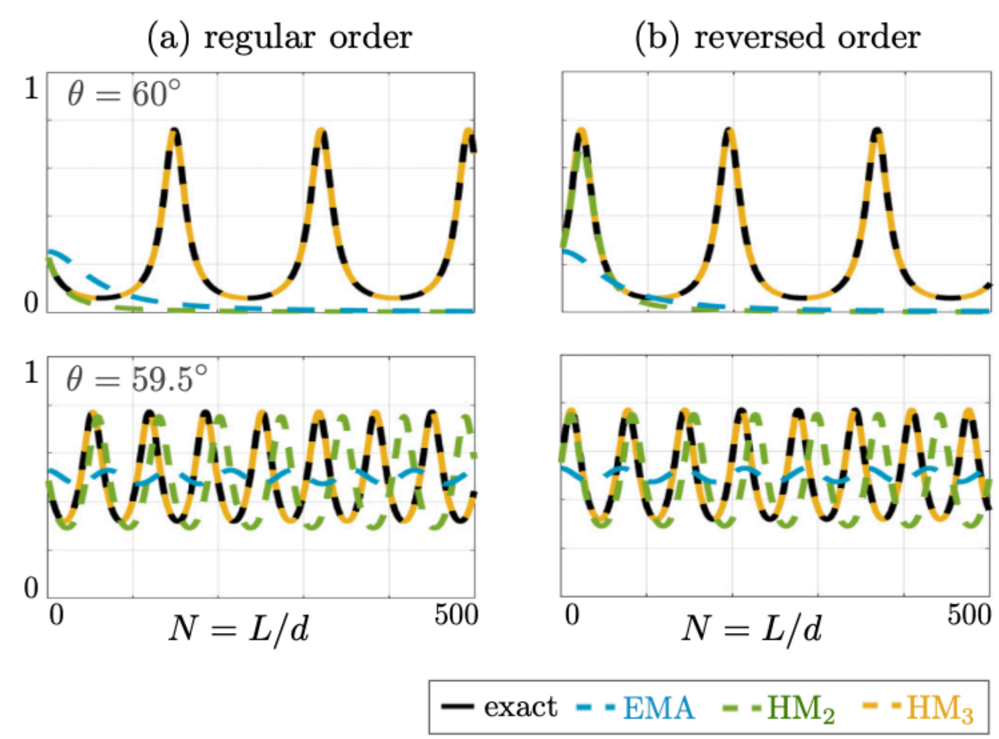
(Received 26 May 2014; published 11 December 2014)



PHYSICAL REVIEW B **98**, 024306 (2018)

Sensitivity of a dielectric layered structure on a scale below the periodicity: A fully local homogenized model

Agnès Maurel[1] and Jean-Jacques Marigo[2]
[1]*Institut Langevin, CNRS UMR 7587, ESPCI Paris, PSL Research University, 1 rue Jussieu, 75005, Paris - France*
[2]*Laboratoire de Mécanique du Solide, Ecole Polytechnique, Route de Saclay, Palaiseau 91120, France*



Metamaterials

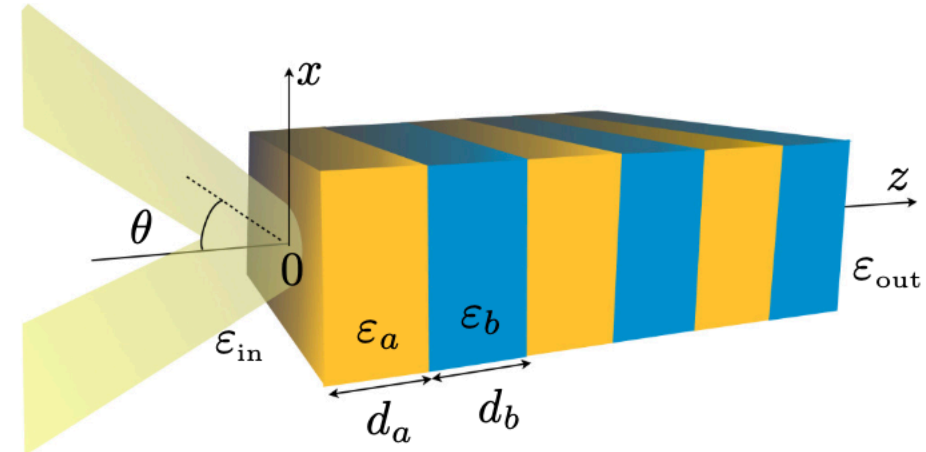
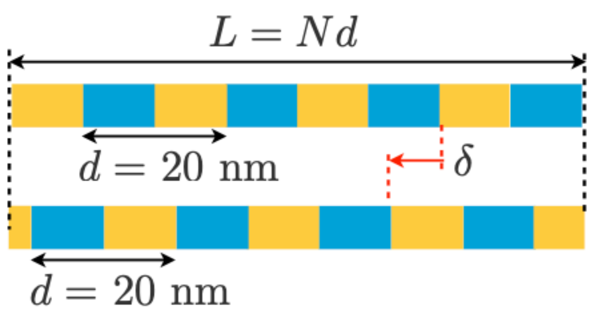
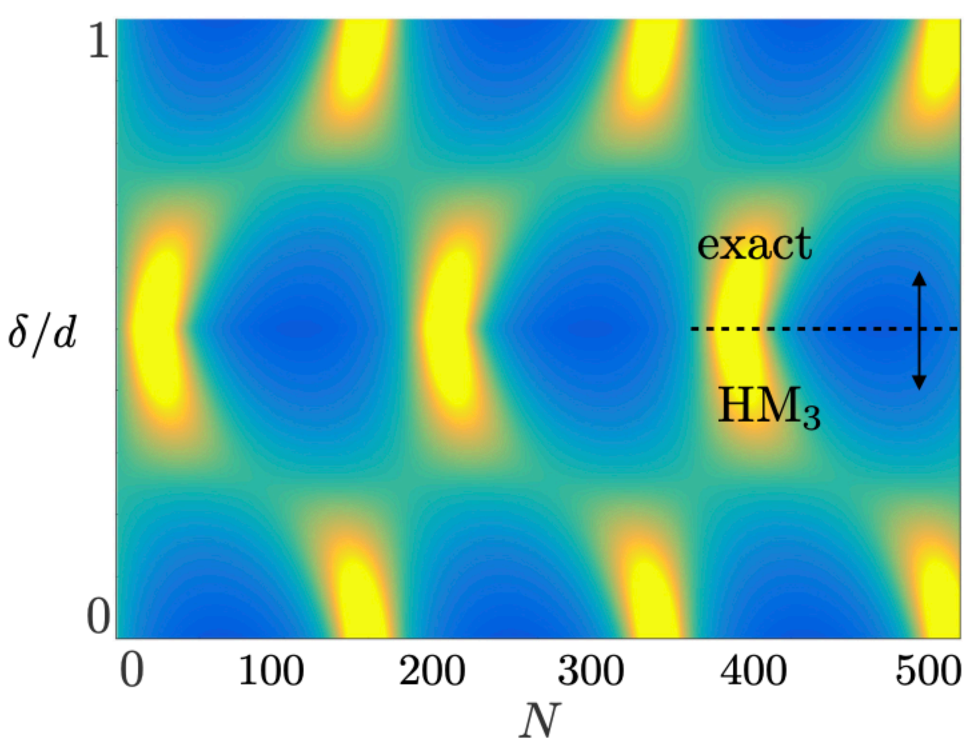


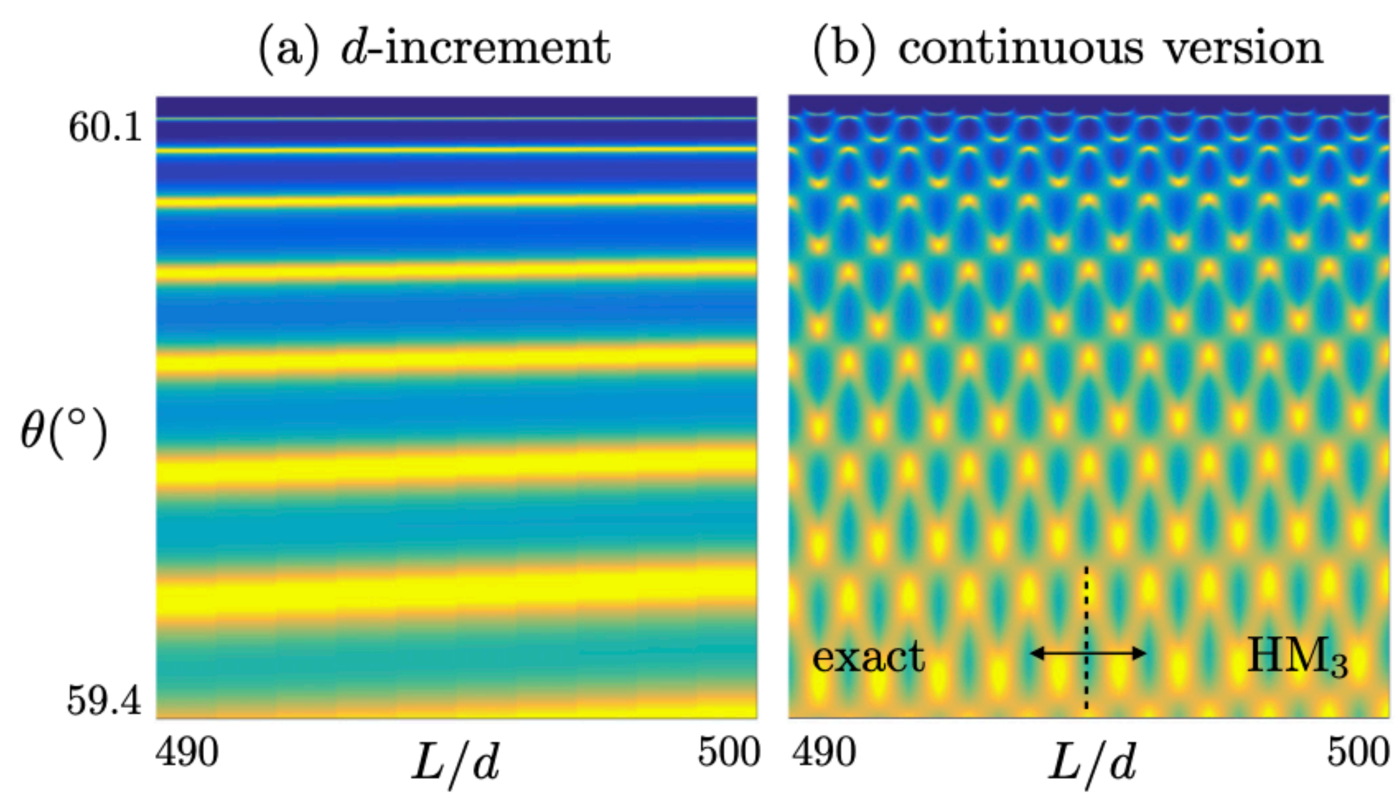
FIG. 1. Multilayer structure illuminated by a plane wave near the critical angle of total reflection ($d = d_a + d_b$ is the period).

Au delà de l'ordre dominant
L'anomalie de Segev

Effets de taille finie



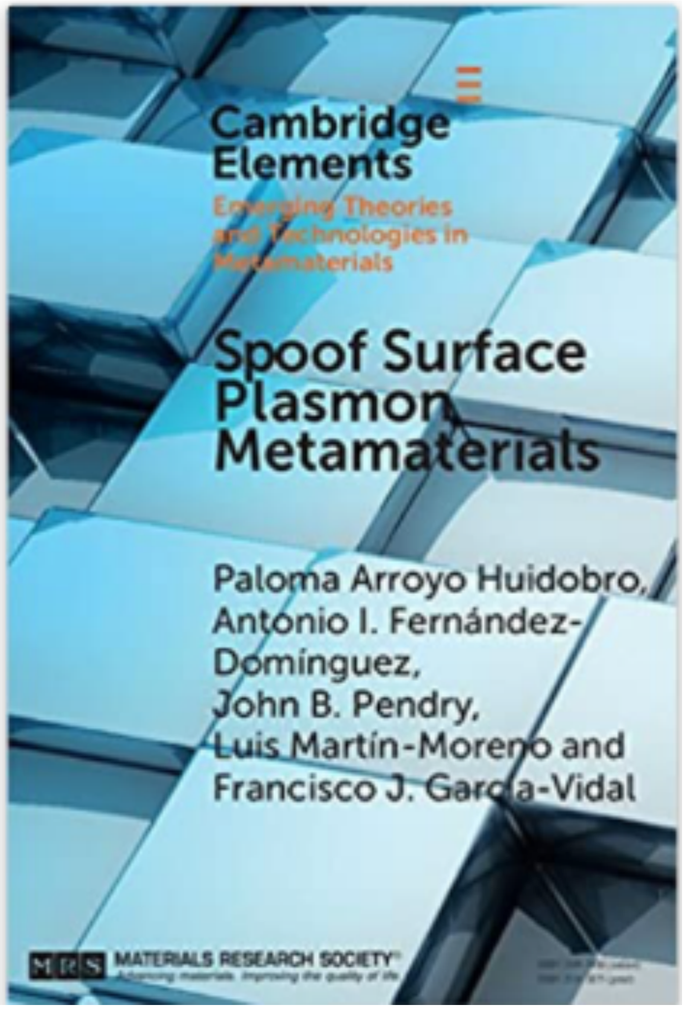




Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials



Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

Surface plasmon polaritons are surface electromagnetic waves that propagate along planar interfaces with sign-changing permittivities.
Spoof surface plasmons are a type of surface plasmon polariton, which ordinarily propagate along metal and dielectric interfaces in infrared and visible frequencies.

Since surface plasmon polaritons cannot exist naturally in microwave and terahertz frequencies due to dispersion properties of metals, spoof surface plasmons necessitate the use of artificially-engineered metamaterials.

Spoof surface plasmons share the natural properties of surface plasmon polaritons, such as dispersion characteristics and subwavelength field confinement.

Wikipedia

Mimicking Surface Plasmons with Structured Surfaces

J. B. Pendry,[1*] L. Martín-Moreno,[2] F. J. Garcia-Vidal[3]

Metals such as silver support surface plasmons: electromagnetic surface excitations localized near the surface that originate from the free electrons of the metal. Surface modes are also observed on highly conducting surfaces perforated by holes. We establish a close connection between the two, showing that electromagnetic waves in both materials are governed by an effective permittivity of the same plasma form. The size and spacing of holes can readily be controlled on all relevant length scales, which allows the creation of designer surface plasmons with almost arbitrary dispersion in frequency and in space, opening new vistas in surface plasmon optics.

Can we understand this gibberish ?

Metamaterials

- Application of the asymptotic homogenization

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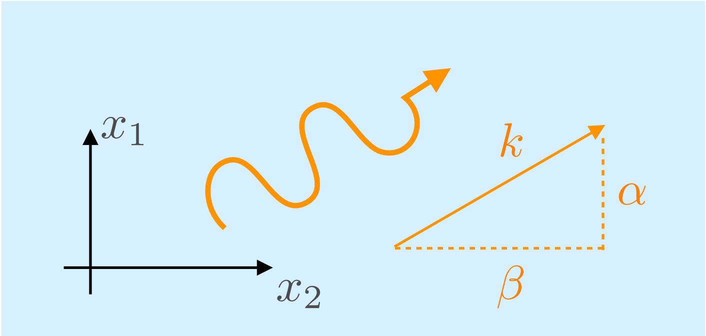
Surface wave : solution of the Helmholtz equation $\Delta \hat{p} + k^2 \hat{p} = 0, \quad k = \frac{\omega}{c}.$

- in an unbounded medium $\hat{p}(\boldsymbol{x}) = e^{i\alpha x_1}\, e^{i\beta x_2}, \quad \alpha^2 + \beta^2 = k^2.$

propagating wave $\beta < k$



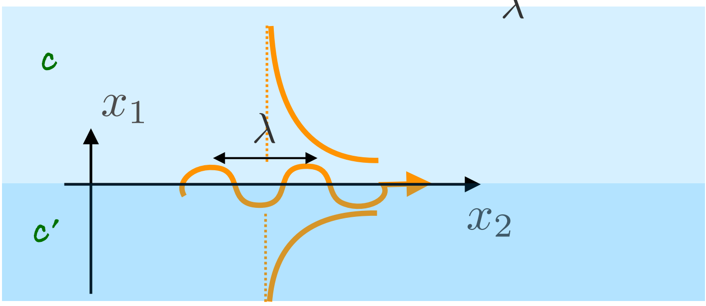
- in two media with an interface

$$\hat{p}(\boldsymbol{x}) = A e^{-\sigma x_1}\, e^{i\beta x_2}, \quad x_1 > 0$$
$$\hat{p}(\boldsymbol{x}) = A' e^{\sigma' x_1}\, e^{i\beta x_2}, \quad x_1 < 0$$

surface wave

$$-\sigma^2 + \beta^2 = k^2, \quad -\sigma'^2 + \beta^2 = k'^2,$$
$$\sigma > 0, \sigma' > 0, \beta \text{ real}$$

$\beta = \frac{2\pi}{\lambda}$



Metamaterials

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surface wave

- in two media with an interface

$$k = \frac{\omega}{c}, \quad k' = \frac{\omega}{c'}$$

$$\hat{p}(\boldsymbol{x}) = A e^{-\sigma x_1}\, e^{i\beta x_2}, \quad x_1 > 0$$

$$\hat{p}(\boldsymbol{x}) = A' e^{\sigma' x_1}\, e^{i\beta x_2}, \quad x_1 < 0$$

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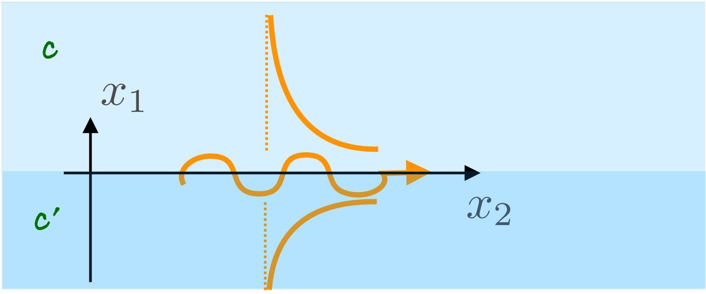

Metamaterials

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$$k = \frac{\omega}{c}, \quad k' = \frac{\omega}{c'}$$

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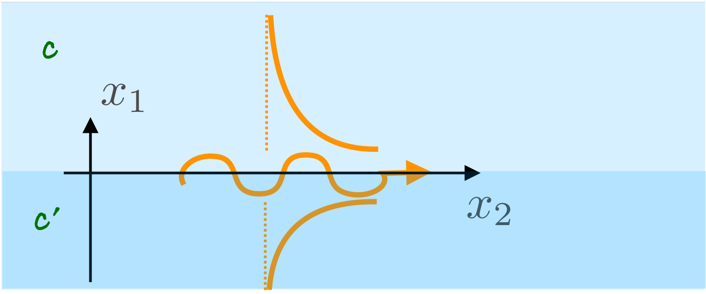

continuity of $\hat{p}$ and $a\nabla\hat{p}\cdot\mathbf{n}$

- continuity relations at $x_1=0$: $\quad Ae^{i\beta x_2} = A'e^{i\beta x_2}, \quad \longrightarrow \quad A = A',$

possible only in electromagnetism

and with $\quad a\nabla\hat{p}\cdot\boldsymbol{n} = a\dfrac{\partial p}{\partial x_1} \quad \longrightarrow \quad -a\sigma A = a'\sigma' A',$

$$-a\sigma = a'\sigma',$$
$$aa' < 0,$$
$$\varepsilon\varepsilon' < 0,$$

	scalar field *p*	vector field ***u***	material parameter ***a***	material parameter ***b***
acoustics (full 3d)	pressure *p*	velocity ***u***	inverse of mass density ρ	inverse of bulk modulus $B = \rho c^2$
electromagnetism (2d polarized)	out-of-plane magnetic field *H*	auxiliary field linked to the in-plane electric field ***E***	inverse of permittivity ε	permeability of vacuum μ_0
elastodynamics (2d)	out-of-plane velocity *u*	in-plane vector stress σ	shear modulus μ	mass density ρ

Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

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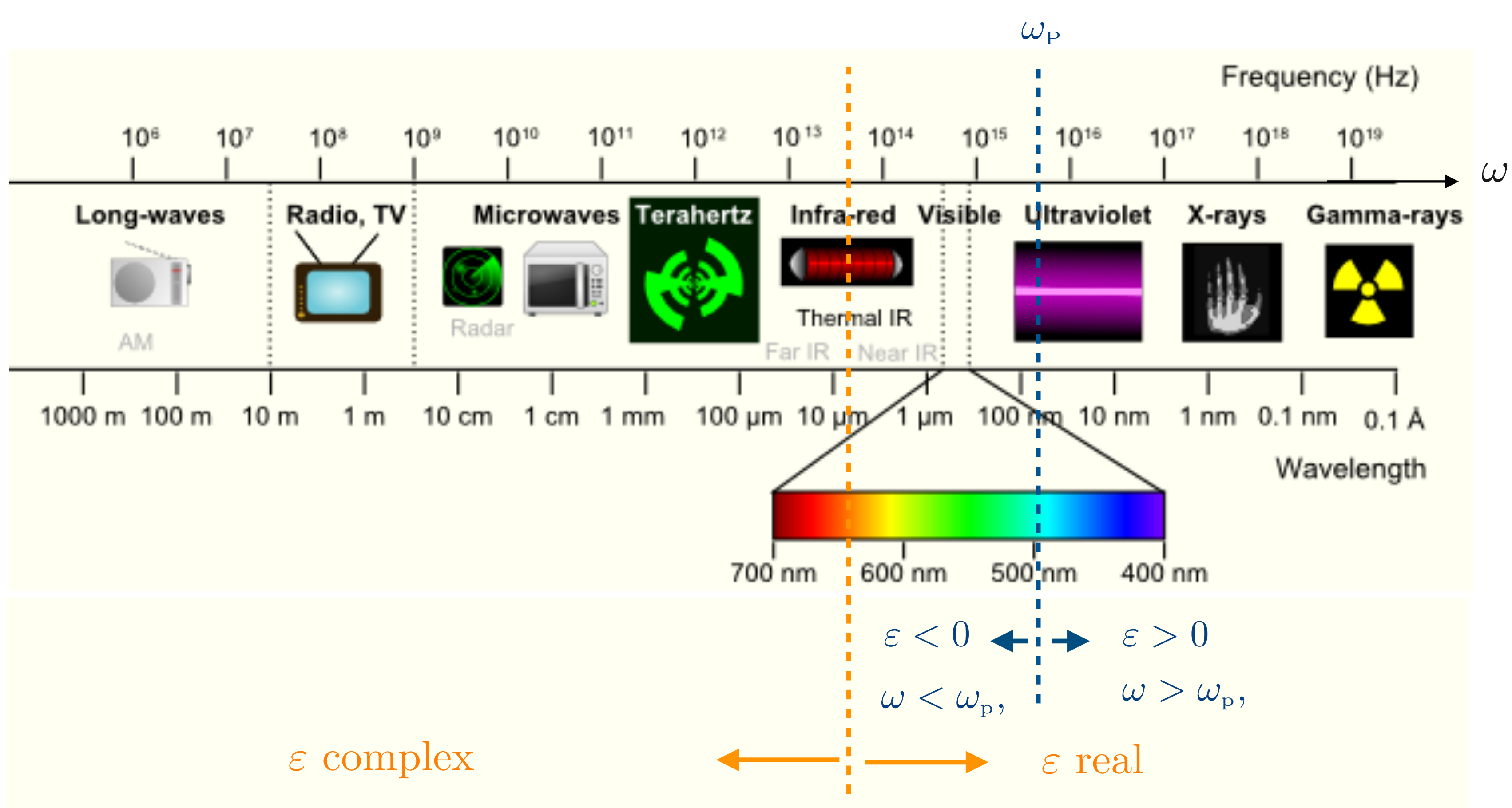
Since surface plasmon polaritons cannot exist naturally in microwave and terahertz frequencies due to dispersion properties of metals, spoof surface plasmons necessitate the use of artificially-engineered metamaterials.

- *metals are conductors and they have a frequency dependent complex permittivity*



Drude law:

$$\varepsilon = \varepsilon_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

ω_p *plasma frequency*

Metamaterials

- Application of the asymptotic homogenization

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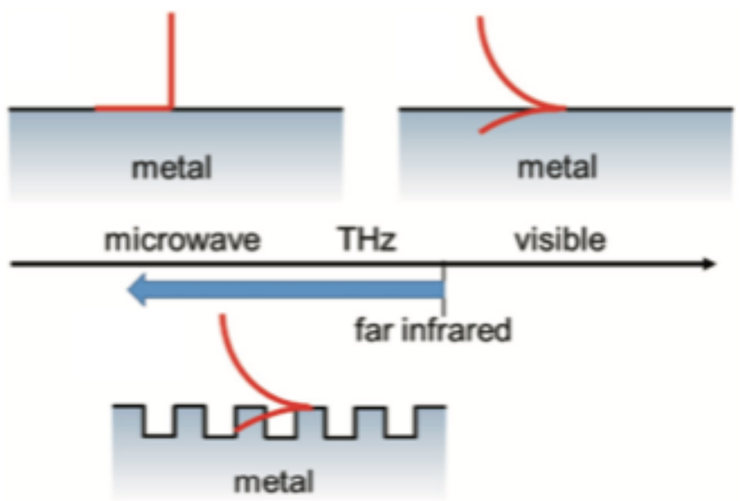
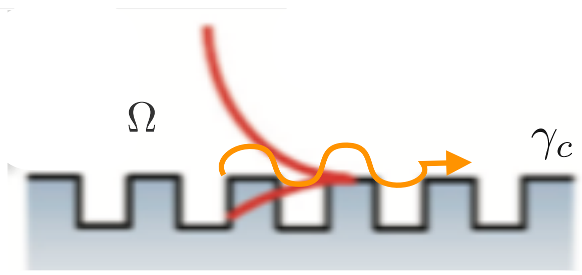


In the 2000s, Pendry and co-workers have shown that a corrugated metallic surface supports guided waves in the far-infrared and microwave, where a metal is associated to Neumann boundary condition.

These guided waves were termed « Spoof Plasmon Polaritons » (SPPs).



$$\longrightarrow k^2\hat{p} + \Delta\hat{p} = 0, \quad k = \frac{\omega}{c}.$$

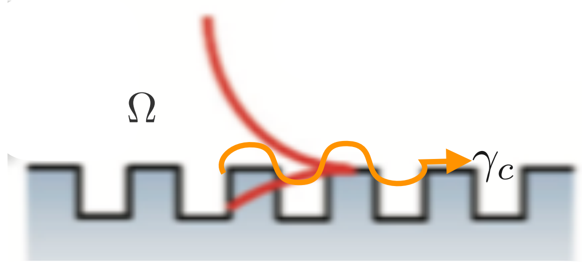
$$\nabla\hat{p}\cdot\mathbf{n}_{|\gamma_c} = 0.$$

can we use the homogenization to model this structure ?

Metamaterials

- Application of the asymptotic homogenization

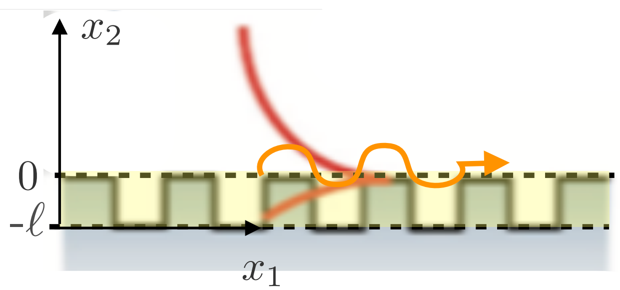
Spoof surface plasmons (SPPs) metamaterials



actual problem

$$\longrightarrow k^2\hat{p} + \Delta\hat{p} = 0, \quad k = \frac{\omega}{c} = \omega\ (c = 1)$$

$$\nabla\hat{p}\cdot\mathbf{n}_{|\gamma_c} = 0.$$



effective problem

conditions at $x_2 = -\ell, 0$?

$$\frac{\partial^2\hat{P}}{\partial x_2^2} + k^2\hat{P} = 0, \quad x_2 \in (-\ell, 0), \qquad \hat{U}_2 = \varphi\frac{\partial\hat{P}}{\partial x_2},$$

$$\Delta\hat{P} + k^2\hat{P} = 0, \quad x_2 \in (0, +\infty), \qquad \hat{U}_2 = \frac{\partial\hat{P}}{\partial x_2},$$

$\hat{P}$, $\hat{U}_2$ continuous at $x_2 = 0$,

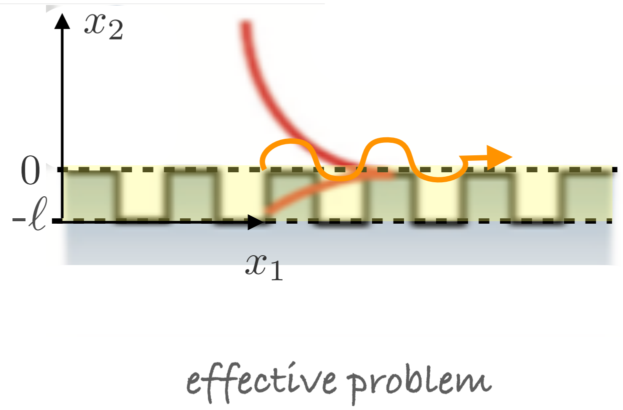
$\hat{U}_2 = 0$ at $x_2 = -\ell$ (Neumann b.c.),

In the effective problem, the dispersion relation of the SPPs can be obtained explicitly

Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials



$$\frac{\partial^2 \hat{P}}{\partial x_2^2} + k^2 \hat{P} = 0, \quad x_2 \in (-\ell, 0), \qquad \hat{U}_2 = \varphi \frac{\partial \hat{P}}{\partial x_2},$$

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$\hat{P}$, $\hat{U}_2$ continuous at $x_2 = 0$,
$\hat{U}_2 = 0$ at $x_2 = -\ell$ (Neumann b.c.),

- Solution of the form

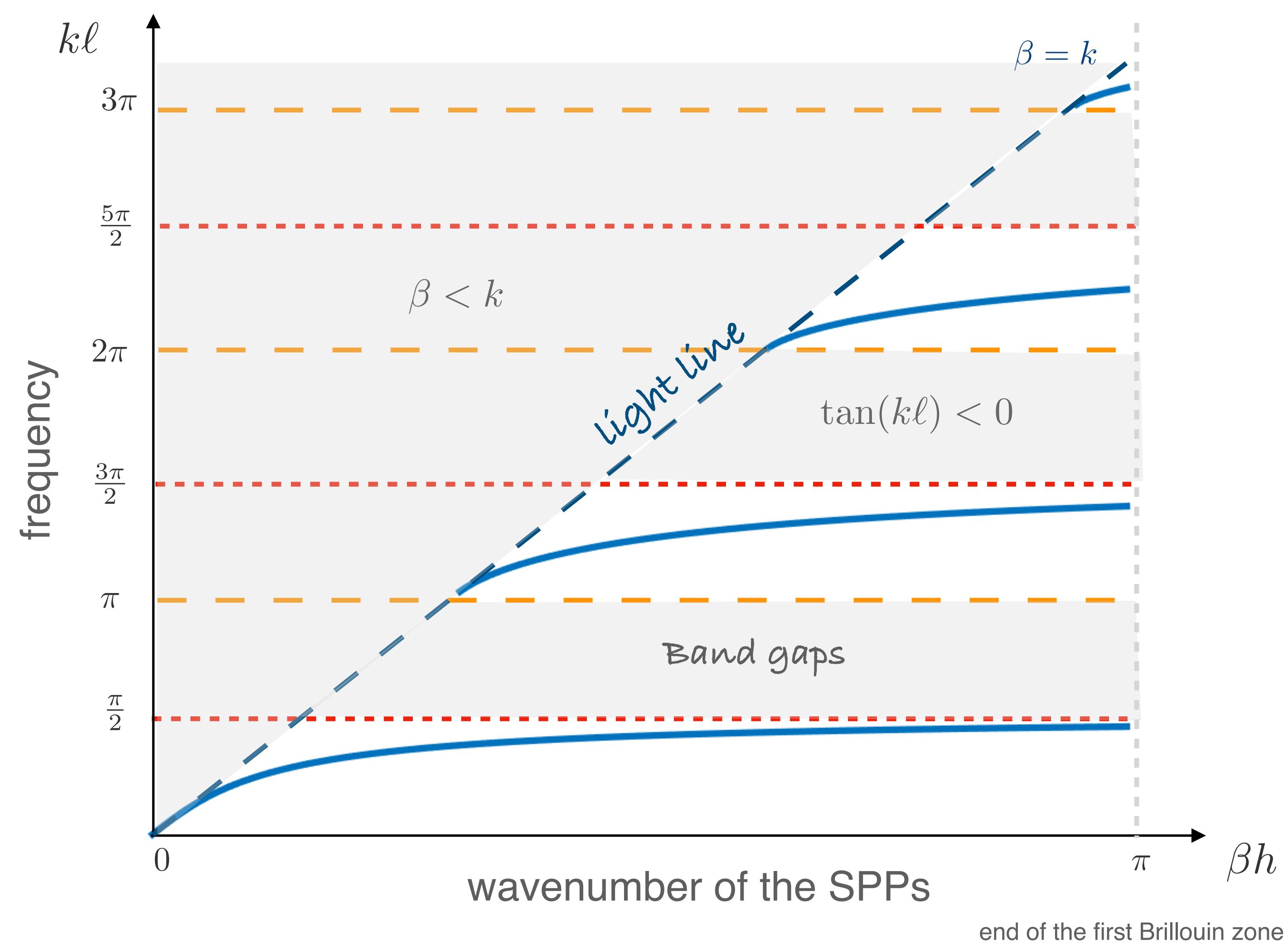
$$\hat{P}(\boldsymbol{x}) = A e^{-\sigma x_2} e^{i\beta x_1}, \qquad x_2 \in (0, +\infty)$$

$$\hat{P}(\boldsymbol{x}) = A \frac{\cos(k(x_2+\ell))}{\cos(k\ell)} e^{i\beta x_1}, \qquad x_2 \in (-\ell, 0)$$

$$\beta = k\sqrt{1 + \varphi^2 \tan^2(k\ell)}.$$

$\tan(k\ell) > 0,$

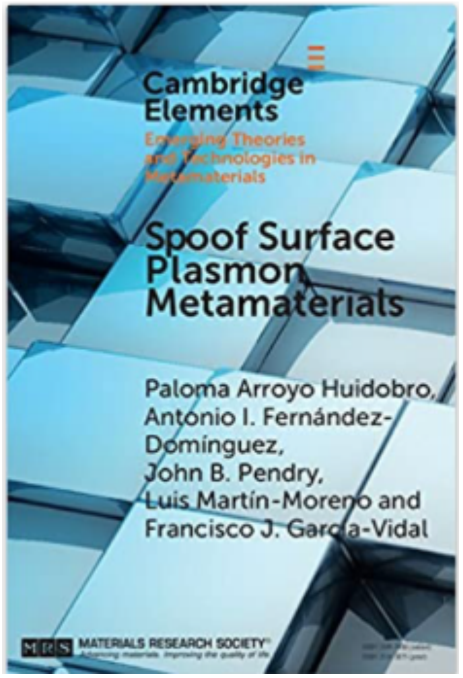
$-\sigma^2 + \beta^2 = k^2, \quad \sigma > 0, \qquad k = \omega/c$



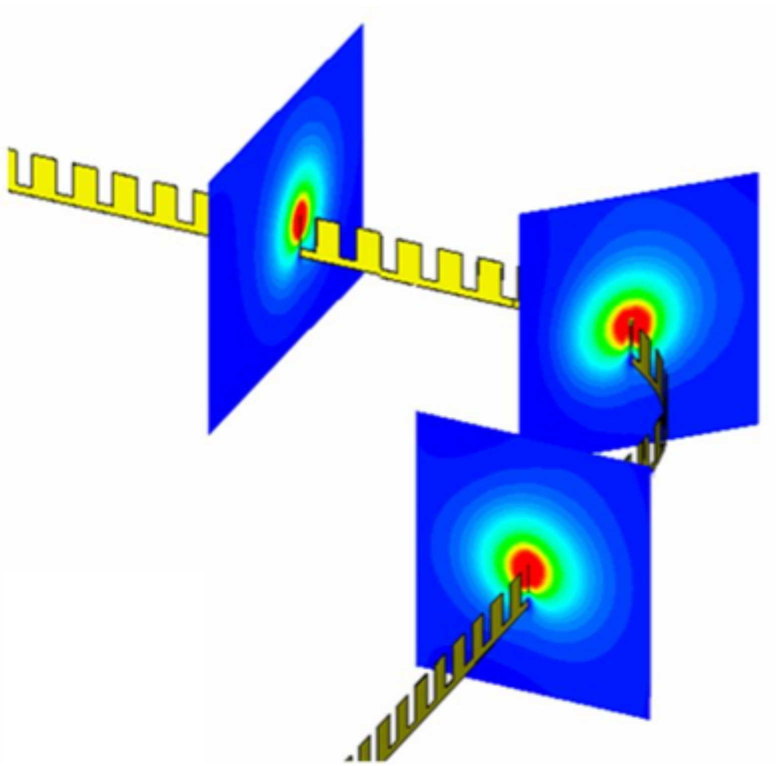
Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials



- As any guided wave, SPPs have being used for application to guiding applicable now to any context of waves (EM, acoustics, elastodynamics) subwavelength guiding as $\beta = k\sqrt{1+\varphi^2\tan^2(k\ell)} > k,$



Metamaterials

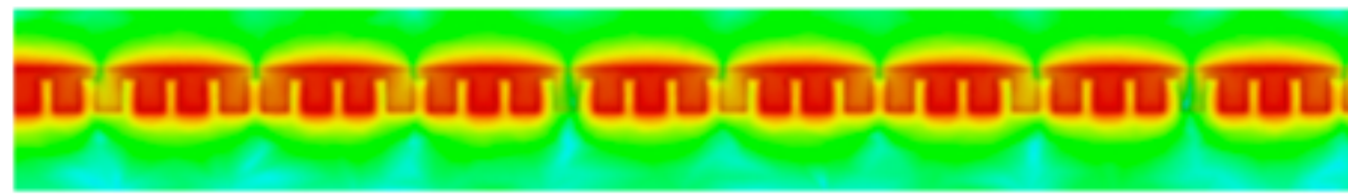
- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

APPLIED PHYSICS LETTERS **111**, 261105 (2017)

Plasma modification of spoof plasmon propagation along metamaterial-air interfaces

R. Lee, B. Wang, and M. A. Cappelli
Stanford Plasma Physics Laboratory, Department of Mechanical Engineering, Stanford University, Stanford, California 94305, USA



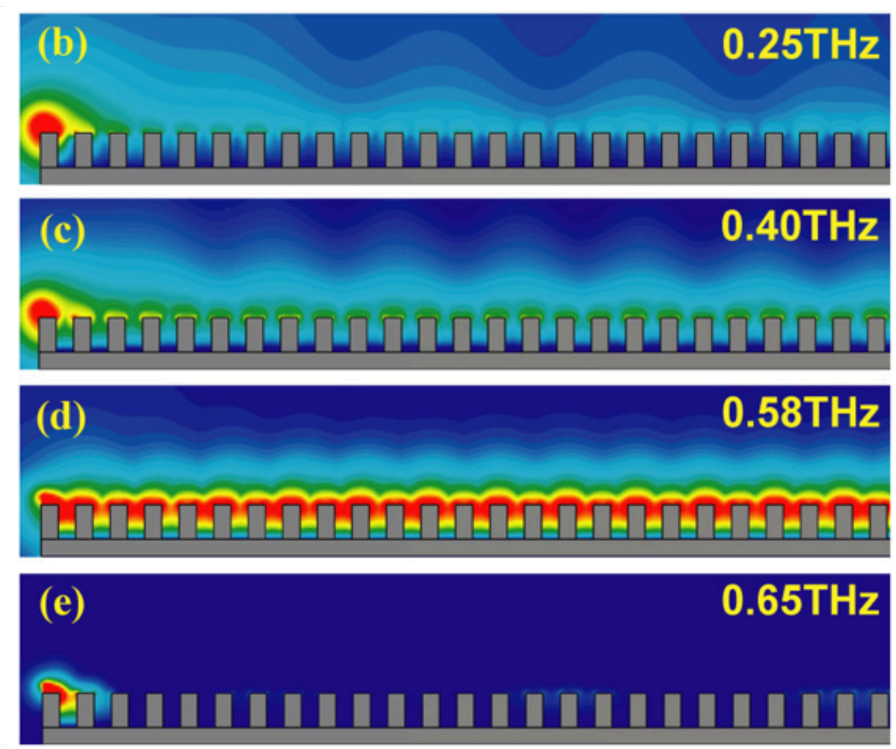
Contents lists available at SciVerse ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Terahertz surface plasmon polaritons in textured metal surfaces formed by square arrays of metallic pillars

Zhen Gao [a], Linfang Shen [a,*], Jin-Jei Wu [b], Tzong-Jer Yang [b], Xiaodong Zheng [c]



electromagnetism

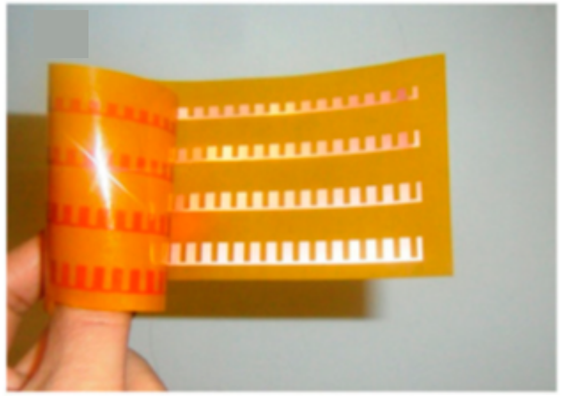
Metamaterials

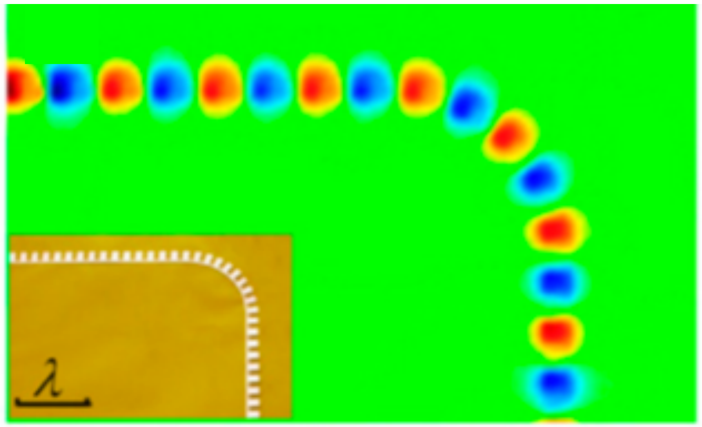
- Application of the asymptotic homogenization

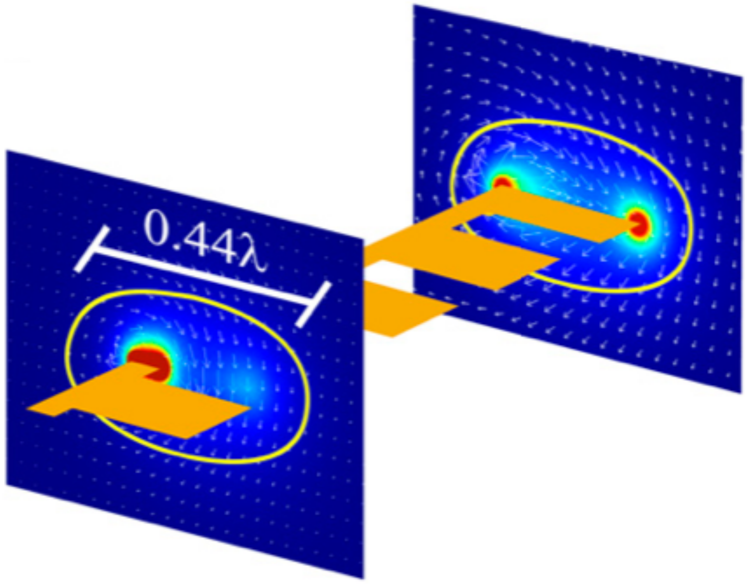
Spoof surface plasmons (SPPs) metamaterials

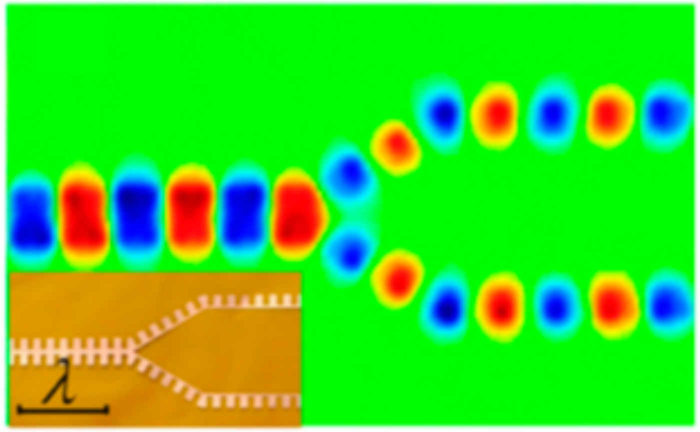
Conformal surface plasmons propagating on ultrathin and flexible films

Xiaopeng Shen[a,1], Tie Jun Cui[a,1,2], Diego Martin-Cano[b], and Francisco J. Garcia-Vidal[b,c,2]









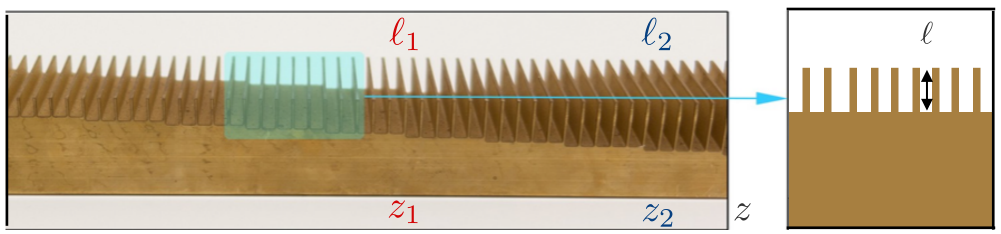
electromagnetism

Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The rainbow effect



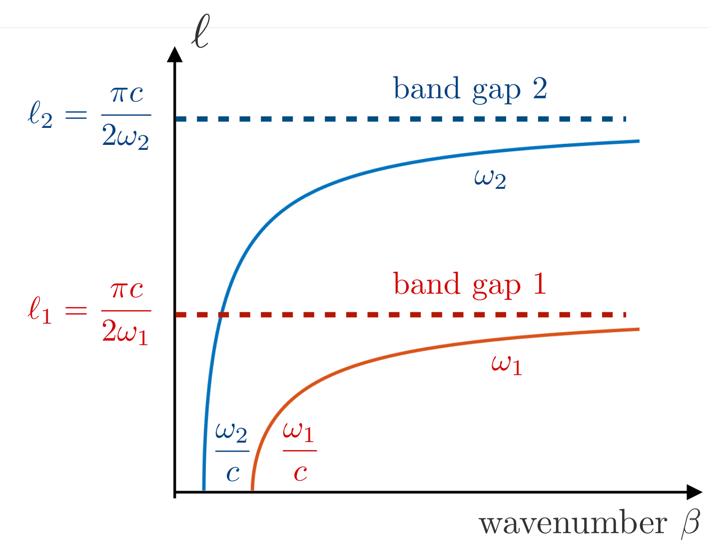
$\ell(z)$ increases as z increases $\quad \ell_2 > \ell_1$

Scientific Reports **3**, Article number: 1728 (2013)

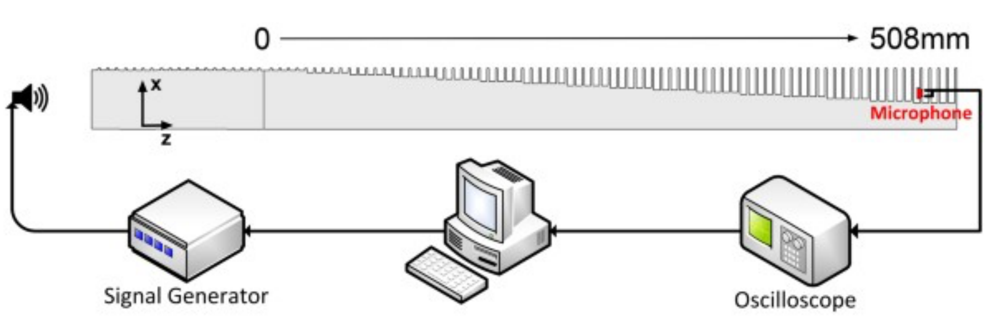
the wave at w_1 can propagate up to $z=z_1$, there its group velocity is almost 0, it is trapped at this position.

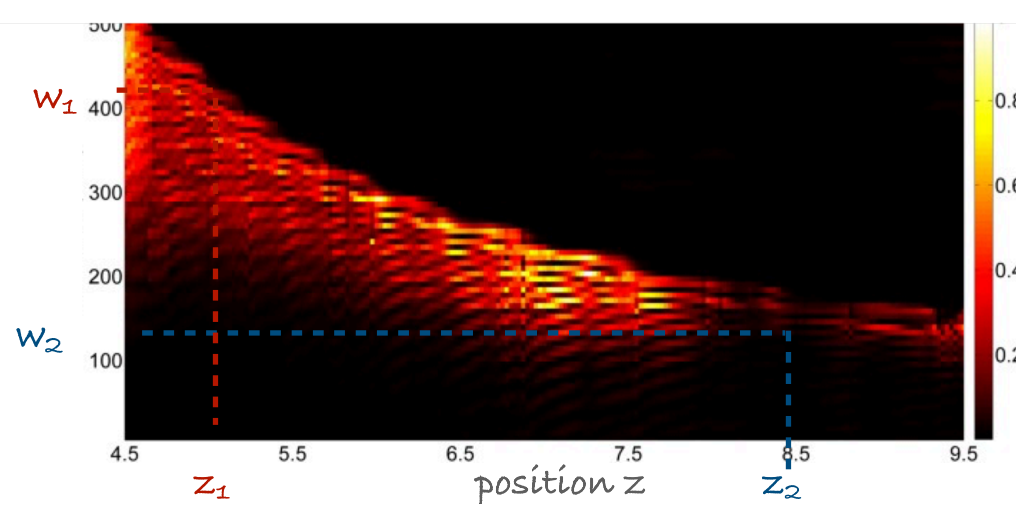
the wave at w_2 can propagate up to $z=z_2$, there its group velocity is almost 0, it is trapped at this position.

If the signal contains a large frequency band (white light), each frequency is trapped at a given position (rainbow). higher frequencies are trapped before lower ones.



$$\beta(\omega, z) = \frac{\omega}{c}\sqrt{1 + \varphi^2 \tan^2\left(\frac{\omega \ell(z)}{c}\right)}. \quad \tan\left(\frac{\omega \ell(z)}{c}\right) > 0,$$





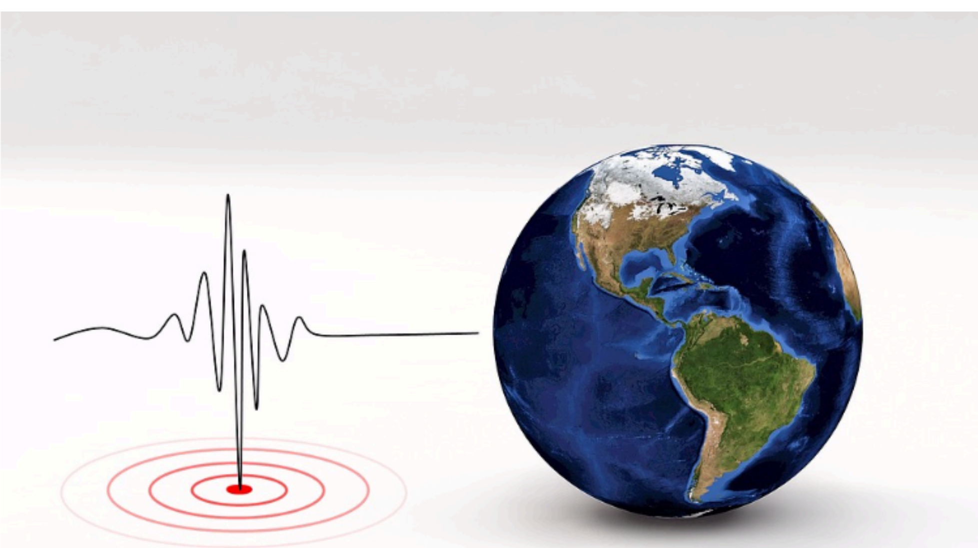
Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

surface waves propagate
on the surface of the Earth under usual conditions



1985 Mexico City (epicenter: 400 km)

Metamaterials

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surface waves propagate
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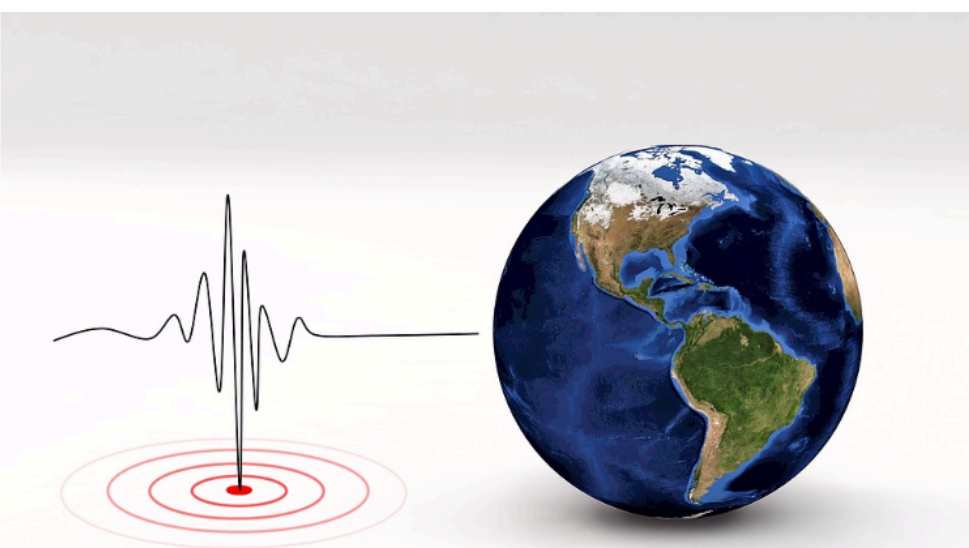
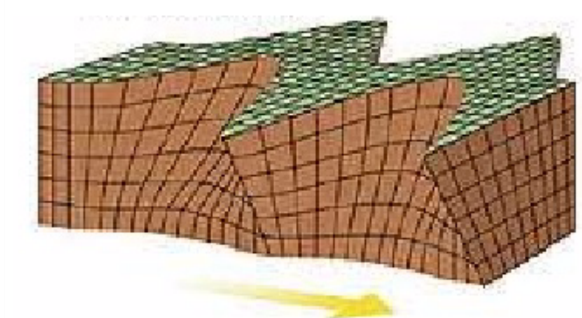


- Love waves (anti-plane displacements)



- Rayleigh waves (in-plane displacements)

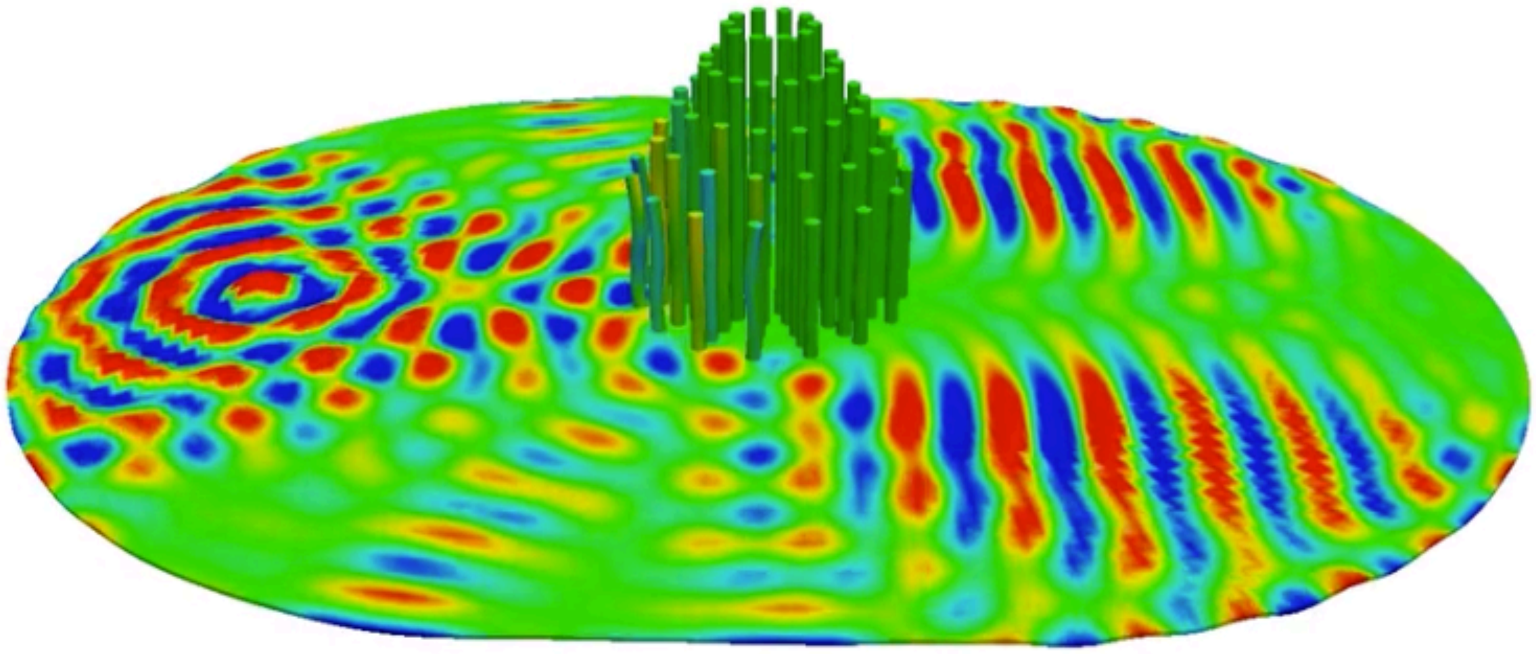
Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

a protection strategy is to deflect the waves from the area to be protected
it is not very nice for the surrounding regions



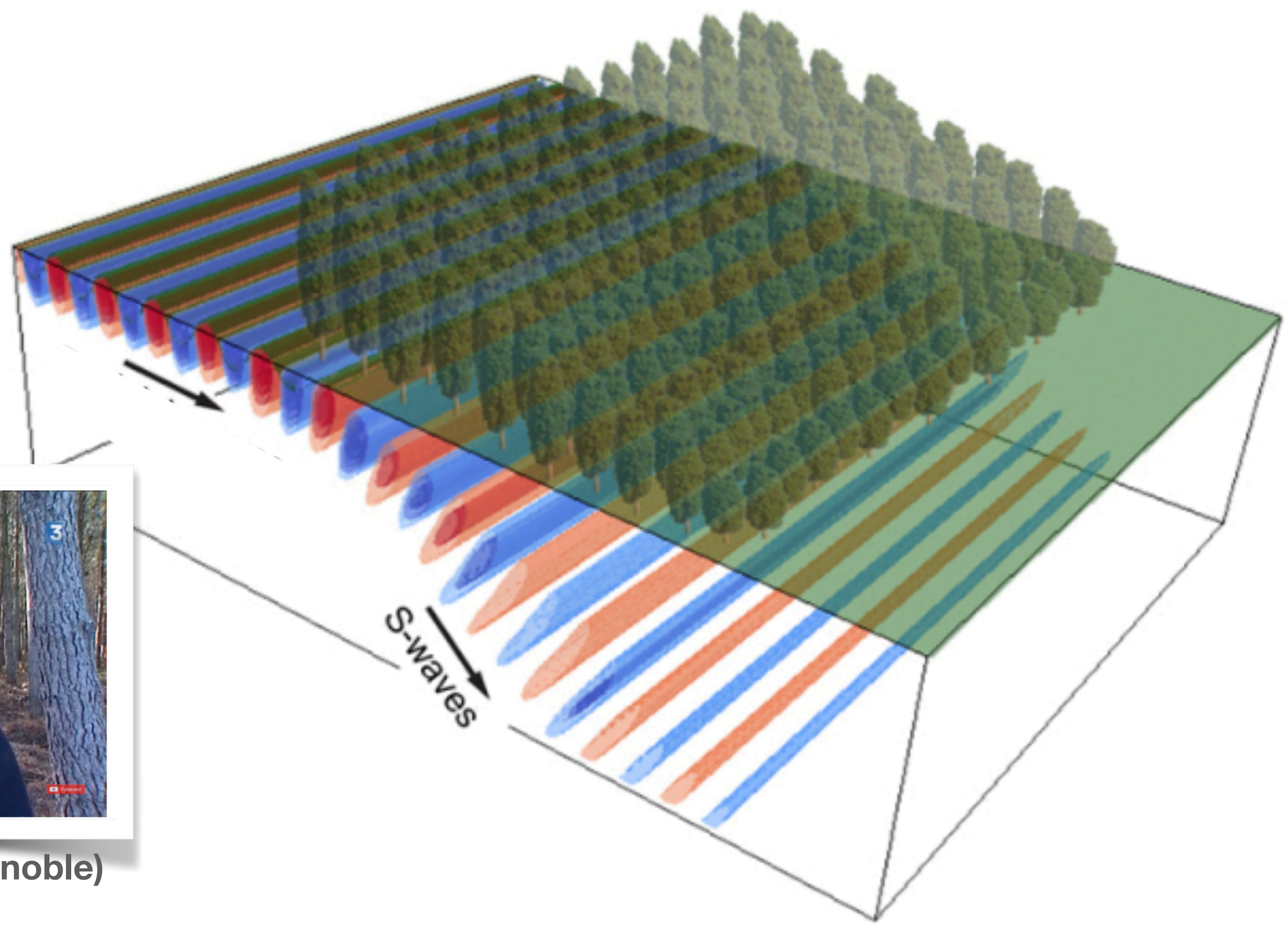
Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

a different strategy consists in converting the seismic waves into downward-propagating bulk S-waves



Philippe Roux (ISTerre - Grenoble)

PR: Seismic metamaterials are materials – such as soil, rock or another elastic medium – that have small, resonant elements. Metamaterials normally refer to a bulk medium that is structured. It is more precise (and probably easier) to describe our devices as metasurfaces. Different concepts of seismic metasurface could be designed. For example, a seismic metasurface consists of an array of resonators placed on top of the soil or rock (or within it). This could consist, for instance, of an array of pillars or rods placed on the surface. The pillars or rods (or, in the case of the META-FORET project, the trees) have resonances and these resonances can then be used to interfere with the incoming waves (from urban noise, railways, seismic activity, etc.) to redirect or block them.

META-FORET

The META-FORET project

New developments towards seismic metamaterials

https://metaforet.osug.fr

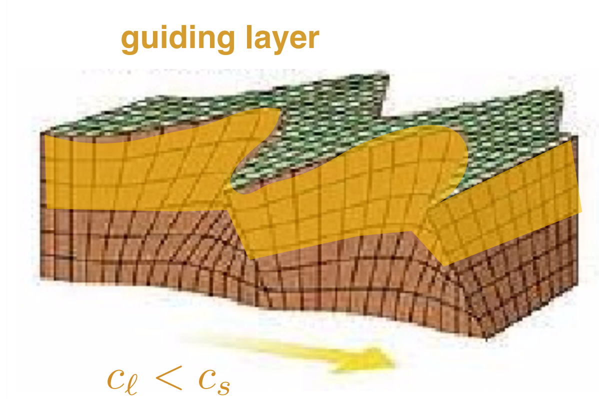
Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

- Love waves (anti-plane displacements)



Love waves can propagate on the free surface of a layered substrate (the Earth)

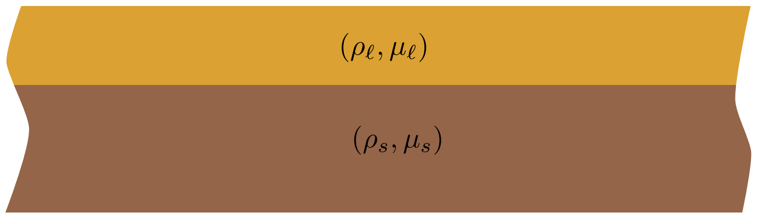
Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

classical Love waves

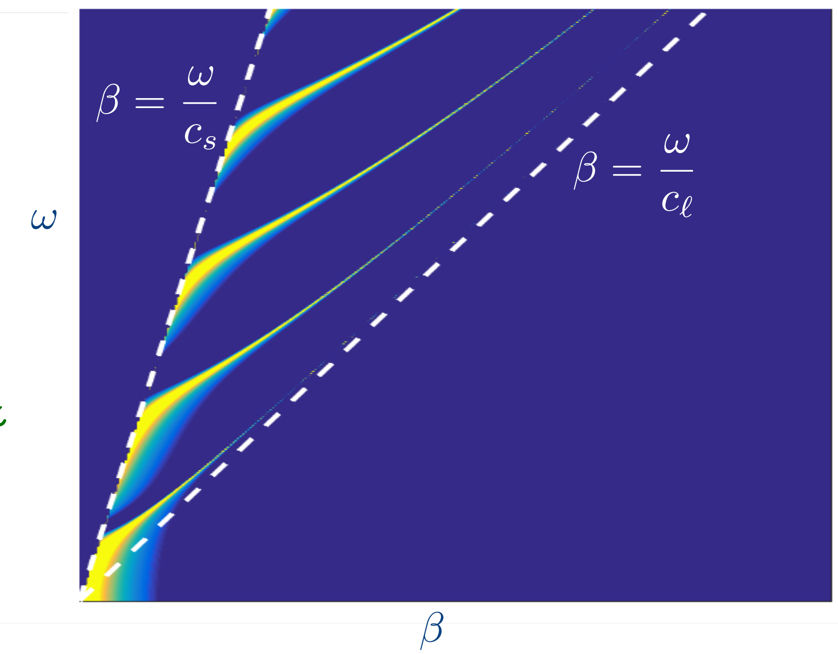


with $k_\ell = \sqrt{\dfrac{\omega^2}{c_\ell^2} - \beta^2}$ $\quad \sigma_s = \sqrt{\beta^2 - \dfrac{\omega^2}{c_s^2}}$

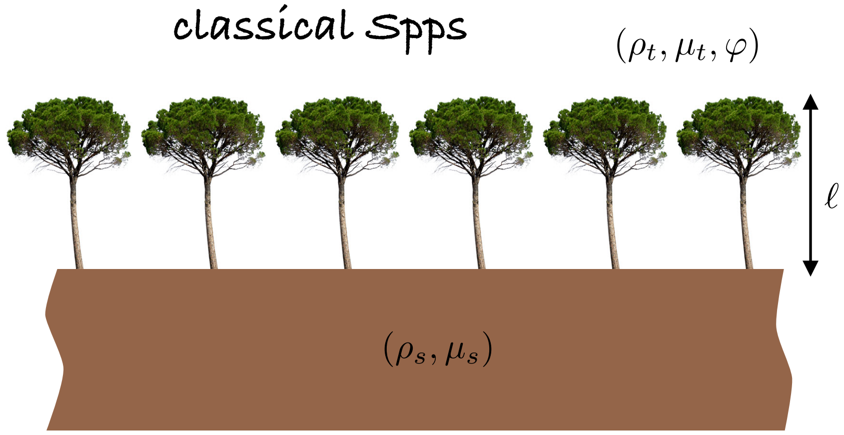
dispersion relation of the Love waves

$$\sigma_s = \frac{\mu_\ell}{\mu_s} k_\ell \tan(k_\ell e)$$

exact



classical Spps

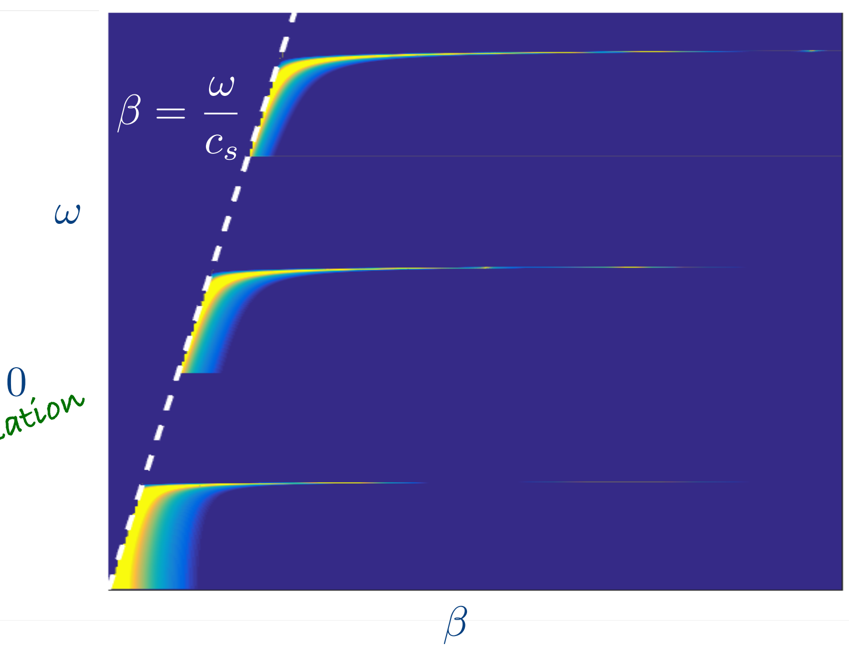


with $k_t = \dfrac{\omega}{c_t}$ $\quad \sigma_s = \sqrt{\beta^2 - \dfrac{\omega^2}{c_s^2}}$

dispersion relation of the elastic SPPs

$$\sigma_s = \frac{\mu_t}{\mu_s} \varphi k_t \tan(k_t \ell) \qquad \tan(k_t \ell) > 0$$

approximate
asymptotic homogenization



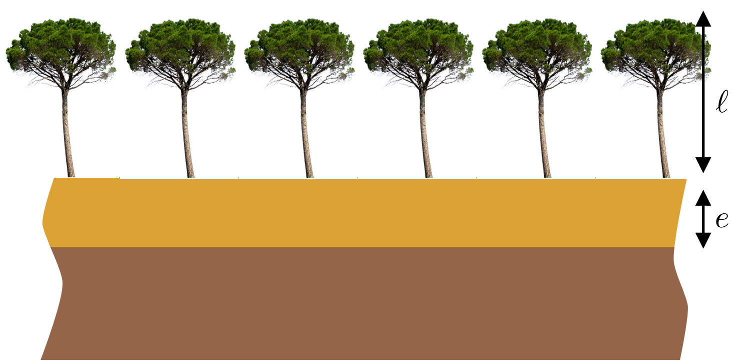
Love waves (anti-plane displacement)

Metamaterials

- Application of the asymptotic homogenization

Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

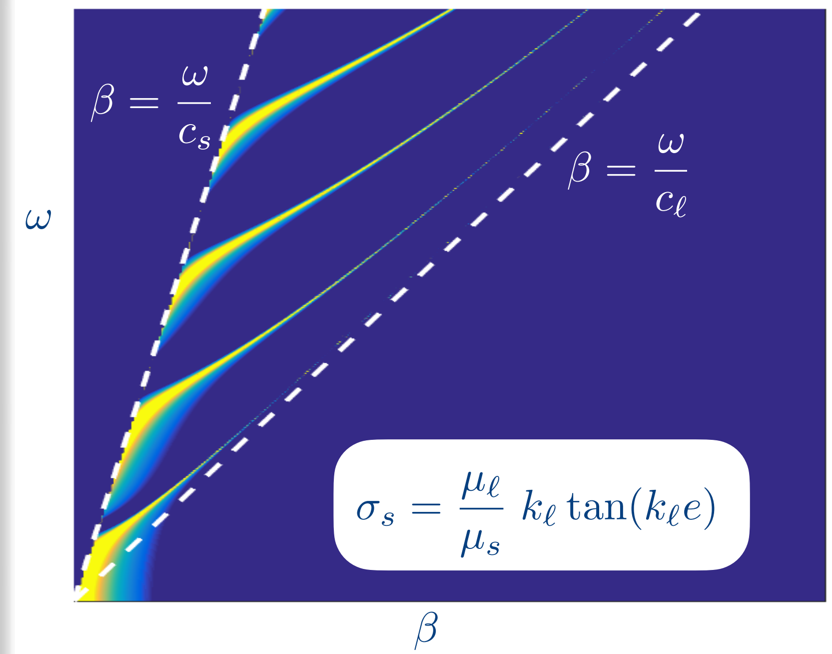


with $k_\ell = \sqrt{\dfrac{\omega^2}{c_\ell^2} - \beta^2}$ $\quad k_t = \dfrac{\omega}{c_t}$ $\quad \sigma_s = \sqrt{\beta^2 - \dfrac{\omega^2}{c_s^2}}$

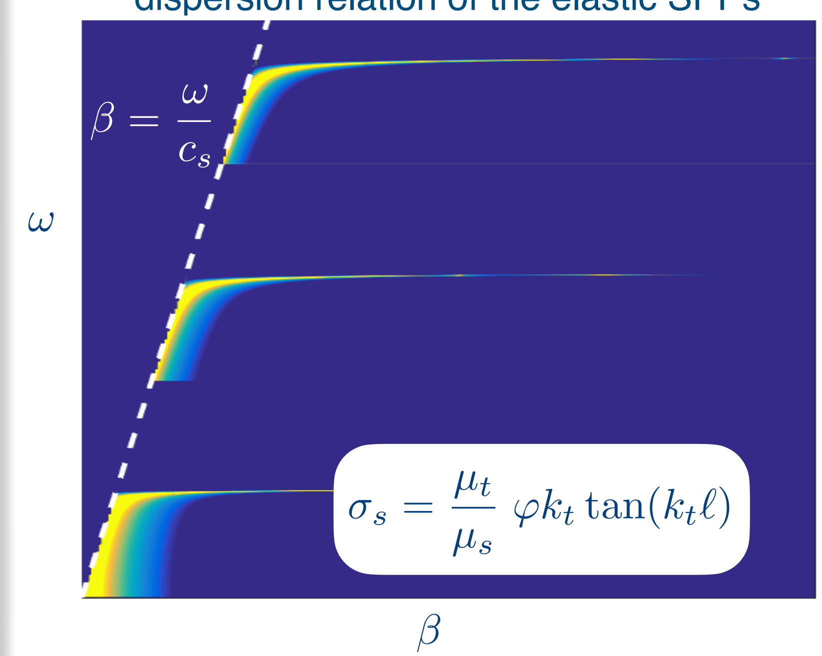
dispersion relation of the Spoof Love waves

$$1 - \frac{\mu_\ell k_\ell}{\mu_s \sigma_s} \tan(k_\ell e) - \frac{\mu_t k_t}{\mu_\ell k_\ell} \varphi \tan(k_t \ell) \left(\tan(k_\ell e) + \frac{\mu_\ell k_\ell}{\mu_s \sigma_s} \right) = 0$$

dispersion relation of the Love waves



dispersion relation of the elastic SPPs



Love waves (anti-plane displacement)

Metamaterials

- Application of the asymptotic homogenization

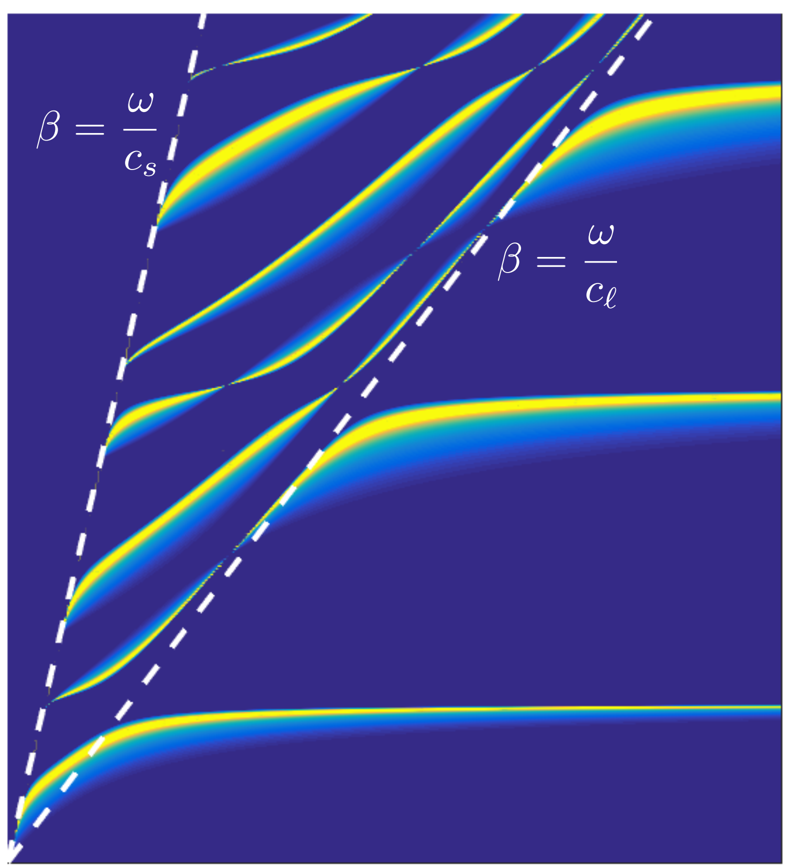
Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves

dispersion relation of the Spoof Love waves

$$1 - \frac{\mu_\ell k_\ell}{\mu_s \sigma_s}\tan(k_\ell e) - \frac{\mu_t k_t}{\mu_\ell k_\ell}\varphi \tan(k_t \ell)\left(\tan(k_\ell e) + \frac{\mu_\ell k_\ell}{\mu_s \sigma_s}\right) = 0$$

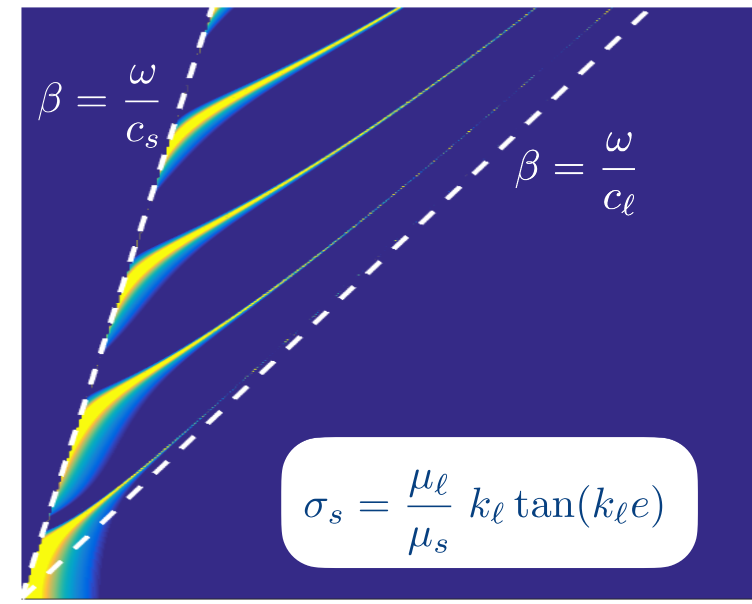
ω



β

dispersion relation of the Love waves

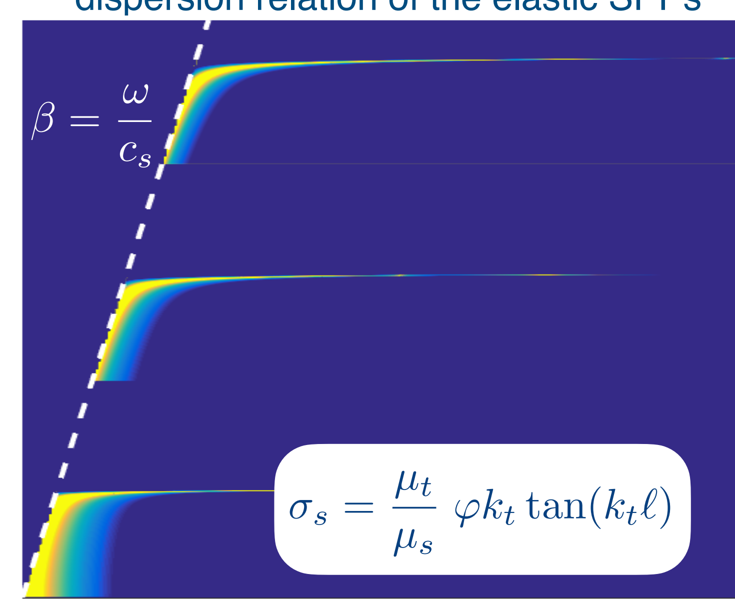
ω



β

dispersion relation of the elastic SPPs

ω



β

Love waves (anti-plane displacement)

Metamaterials

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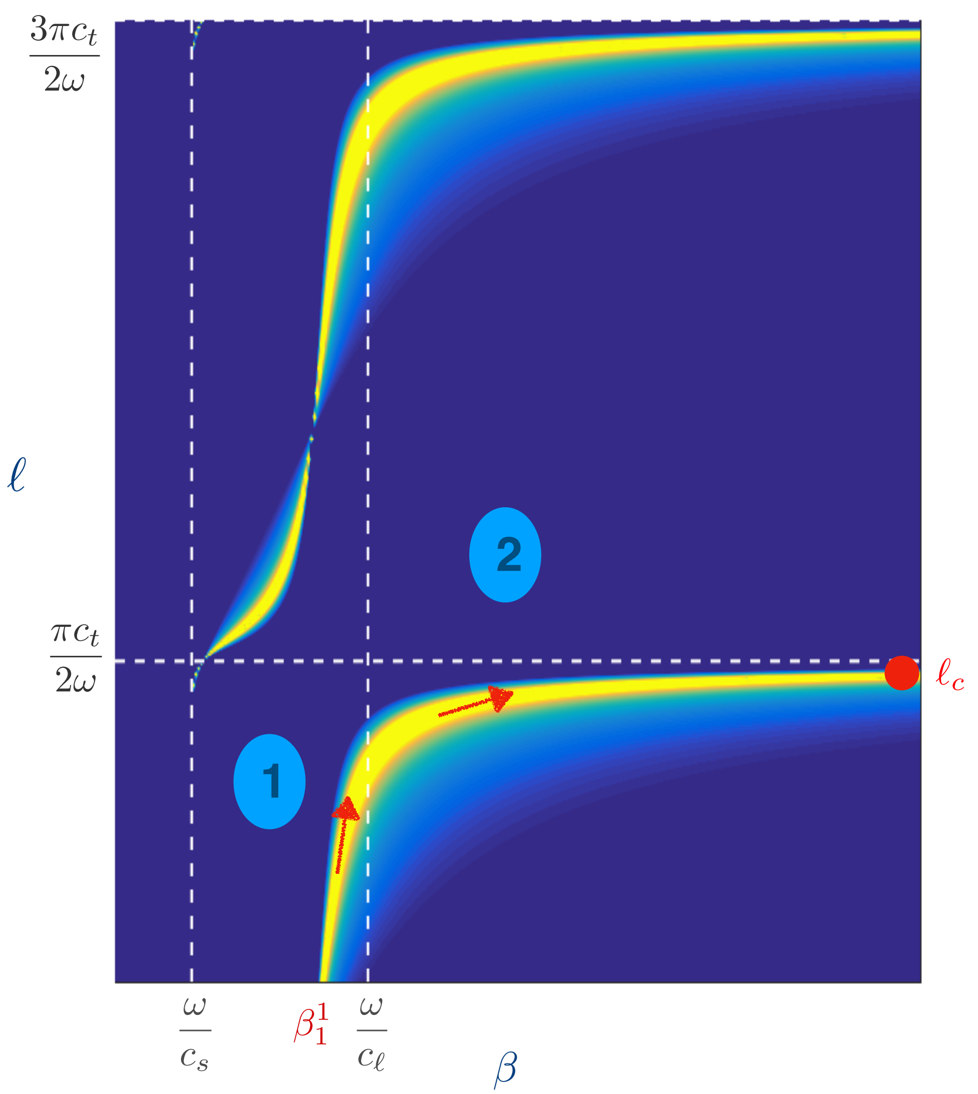
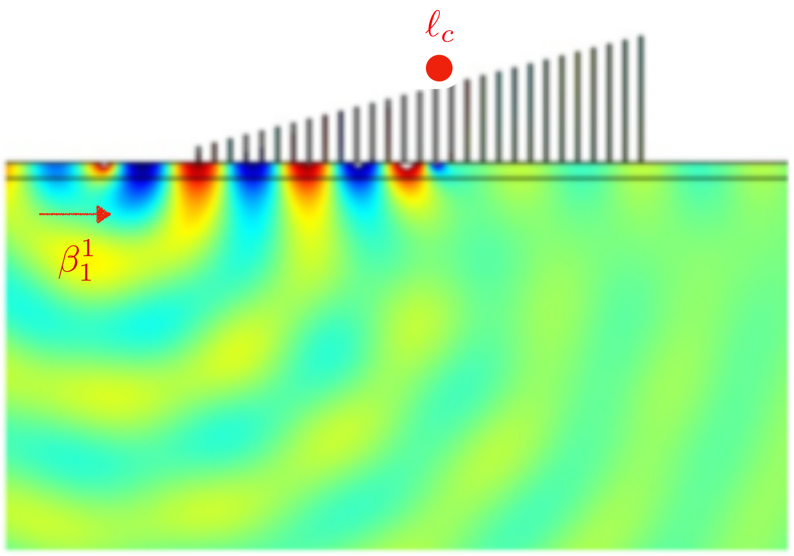
The anti-rainbow effect for the control of seismic waves





classical rainbow effect

First the Love wavenumber increases slightly up to $\frac{\omega}{c_\ell}$

Afterwards, the surface wave, of the Spps type, is supported by the trees only

up to $\ell_c \sim \frac{\pi c_t}{2\omega}$ the wave slows down and it is trapped at some position

The surface wave is always evanescent (along z) in the soil $\beta > \frac{\omega}{c_s}$

1 The surface wave is propagating (along z) in the layer $\beta < \frac{\omega}{c_\ell}$

2 The surface wave is evanescent (along z) in the layer $\beta > \frac{\omega}{c_\ell}$

$$k_\ell = \sqrt{\frac{\omega^2}{c_\ell^2} - \beta^2} \qquad \sigma_s = \sqrt{\beta^2 - \frac{\omega^2}{c_s^2}}$$

Metamaterials

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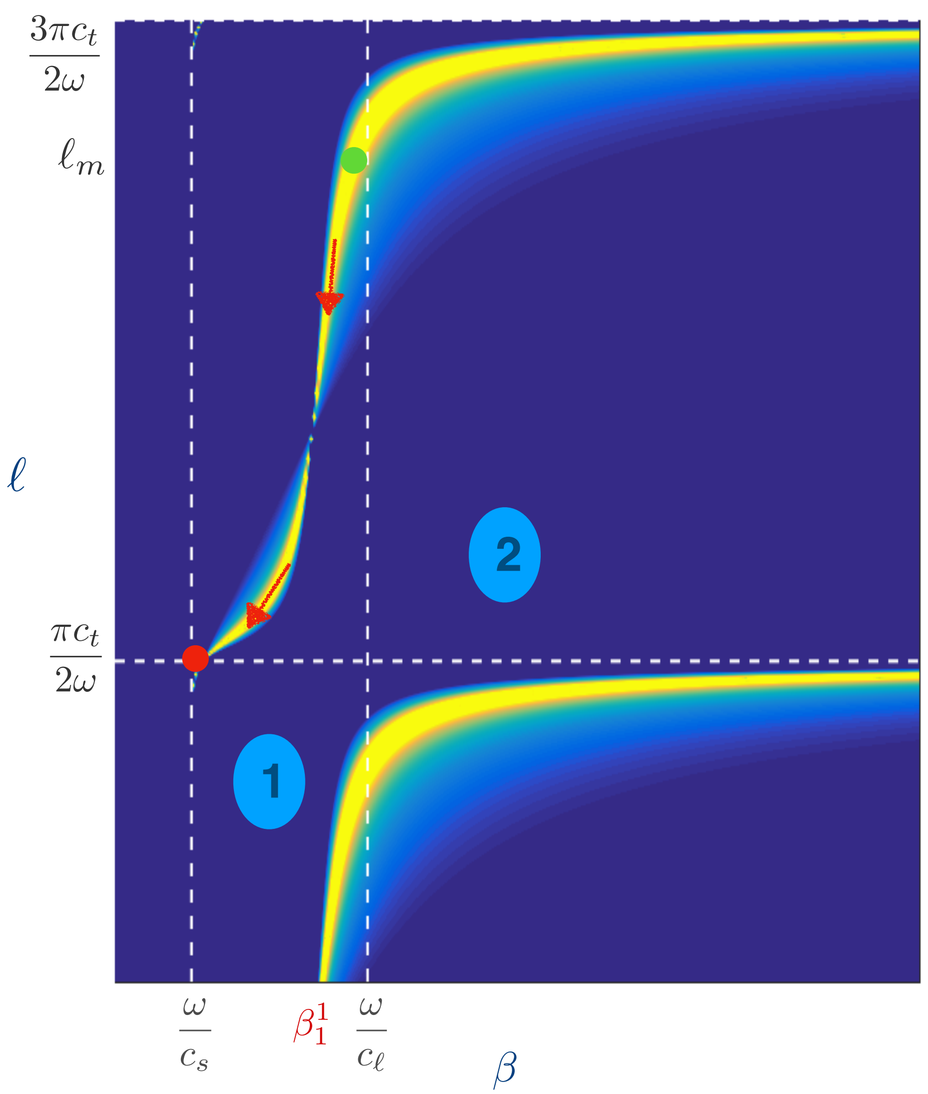
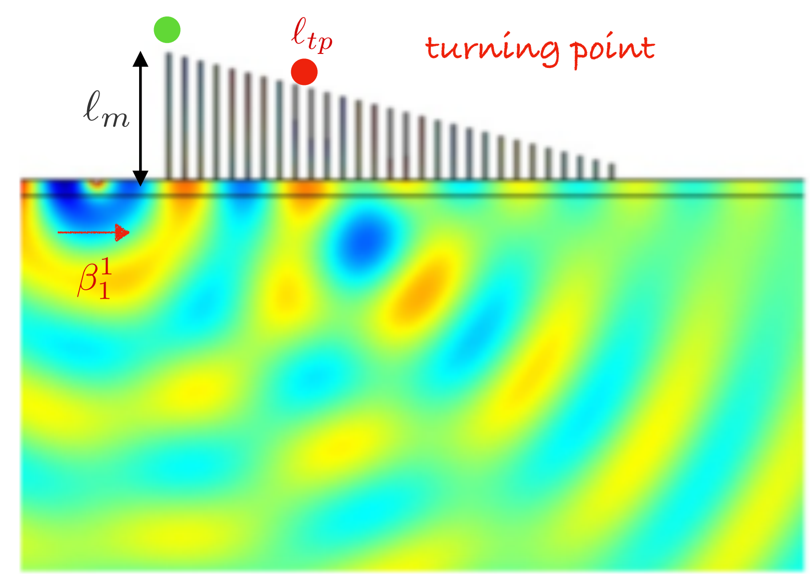
Spoof surface plasmons (SPPs) metamaterials

The anti-rainbow effect for the control of seismic waves





anti-rainbow effect

It is smarter to choose ℓ_m such that $\beta(\ell_m) \sim \beta_1^1$

The wavenumber of the surface wave decreases up to $\ell_{tp} = \frac{\pi c_t}{2\omega}$

There, it is not a surface wave anymore, as $\sigma_s = 0, \beta_s = \frac{\omega}{c_s}$

It is a wave able to propagate within the soil.

The surface wave is always evanescent (along z) in the soil $\beta > \frac{\omega}{c_s}$

1 The surface wave is propagating (along z) in the layer $\beta < \frac{\omega}{c_\ell}$

2 The surface wave is evanescent (along z) in the layer $\beta > \frac{\omega}{c_\ell}$

$$k_\ell = \sqrt{\frac{\omega^2}{c_\ell^2} - \beta^2} \qquad \sigma_s = \sqrt{\beta^2 - \frac{\omega^2}{c_s^2}}$$

Metamaterials

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The anti-rainbow effect for the control of seismic waves

PHYSICAL REVIEW B **98**, 134311 (2018)

Conversion of Love waves in a forest of trees

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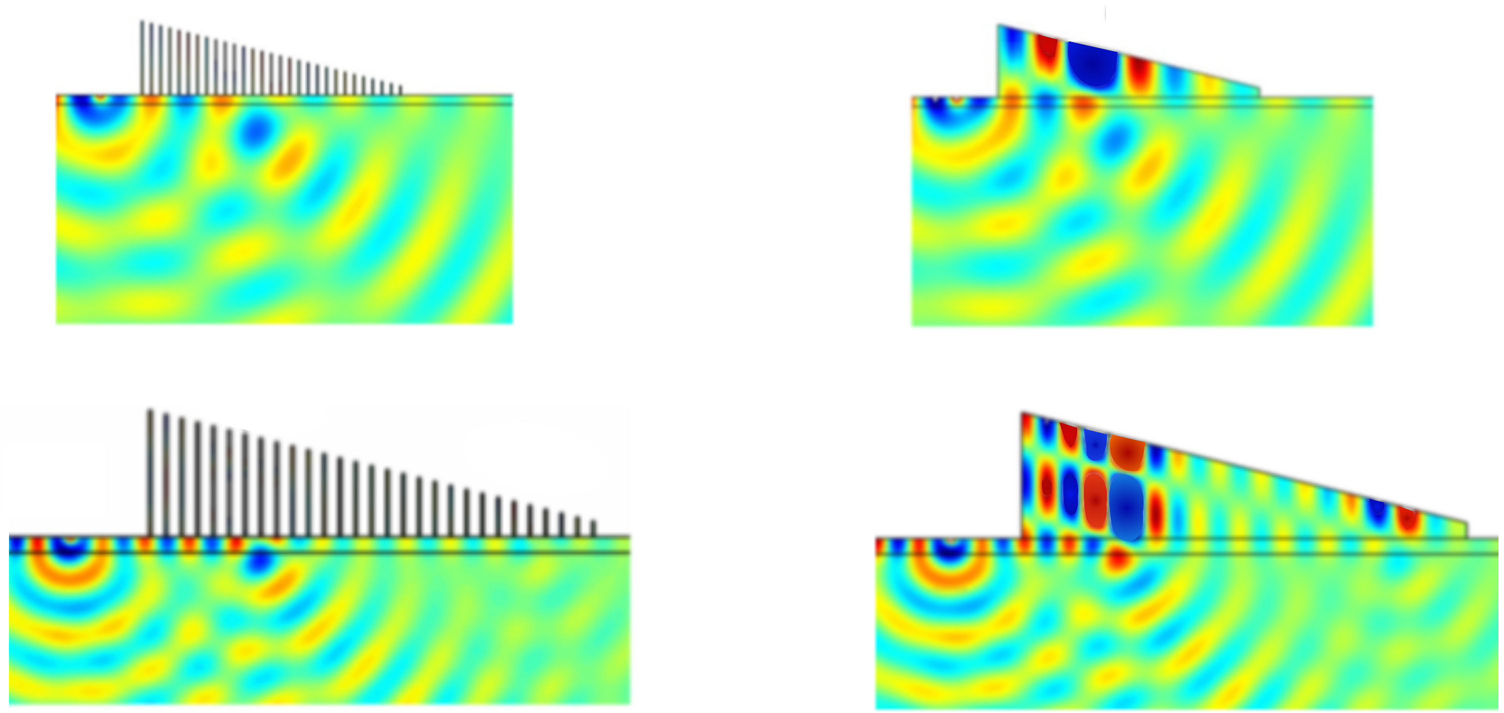
The theoretical analysis is based on asymptotic homogenization

actual solutions

homogenized solutions



1 turning point

2 turning points